Processing of ECG Signals Based on Wavelet Transformation

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Abstract

In this paper present an improved method of ECG delineation approach based on the continuous wavelet transformation and singlescale approach is presented. The algorithm was design to detect nine main ECG significant points, which are QRS onset, QRS pick, QRS offset and analogous points to P wave and T wave. The implemented algorithm improvement significantly increases accuracy of the P wave and T wave detection, which was the problematic part of the previous method. An improved method of wavelet thresholding of ECG signals to remove noise characteristics is provided.

Keywords: Electrocardiogram; P-QRS-T waves; Wavelet Transformation

1. Introduction

An electrocardiogram (ECG) is a recording of electrical potential activity of the heart, taken from one or more leads, and consists of a periodic sequence of cardiac. In a typical cardiac are identified several elements: QRS-complex, P and T waves (Fig. 1). The starting point for a number of modern techniques of a computer electrocardiography is a selection of QRS-complex, which reflects the depolarization of the ventricles. Then, find P and T waves, reflecting the process of depolarization of the atria, and the process of final repolarization of ventricular myocardium [1-3].

![Figure 1. A Typical Complex of ECG Signal](image)

Nowadays to analyze ECG signal it is promising to use wavelet analysis. Wavelets are mathematical functions that are local in time and frequency, and in which all functions are obtained from the same basic functions through its translation and dilation
on the time axis. Compared with the decomposition of signals to Fourier series, wavelets are able to introduce more accurately providing local features of signals [4].

The aim of this work is to improve methods of ECG signals analysis in the high-resolution electrocardiography.

2. Choice the Type of Wavelet

The developed method of ECG signal analysis is based on the continuous wavelet transformation (CWT). CWT at different time scales characterizes the signal in different frequency ranges, while the discrete wavelet transformation (DWT) is limited to scales that are powers of two. Using the CWT gives us more opportunities [4-5].

Wavelet coefficients of a signal in the CWT with a scale factor \(a\) and position \(b\), is defined by

\[
CWT(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} s(t) \psi^* \left( \frac{t-b}{a} \right) dt,
\]

Where \(s\) is a signal and \(\psi\) is a wavelet [4].

To select the optimal wavelet, which is used as a basis, we tested several wavelet functions. The optimal wavelet is called the one that provides the correct location coordinates of nine points of a cardiac cycle: onset, peak and offset T-wave, QRS-complex and P-wave. In [6-7] are used biorthogonal wavelets with compact support, using scales that are multiples of powers of two. In [7-8] are used Gaussian wavelets. The properties of these families of wavelets [4-5] are presented in Table 1.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Gaus</th>
<th>Bior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function (\varphi)</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Function (\psi)</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Orthogonal analysis</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Compact support</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Signal recovery</td>
<td>Recovery is not guaranteed</td>
<td>+</td>
</tr>
<tr>
<td>Symmetry</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>FIR filters</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Possible wavelet analysis</td>
<td>CWT without the use of fast algorithms</td>
<td>CWT and DWT using fast algorithms</td>
</tr>
</tbody>
</table>

The best type of a wavelet, which satisfies the above requirements, is a biorthogonal wavelet "bior1.5". Testing results are presented in Table 2.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Gaus</th>
<th>Bior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“bior1.1”</td>
<td>“bior1.3”</td>
</tr>
<tr>
<td>Accuracy, %</td>
<td>91-92</td>
<td>93-94</td>
</tr>
</tbody>
</table>

To determine the correct location of nine coordinates of ECG signal points was used the 15 scale for detecting QRS-complex [7-11] and the 41 of the scale for the detection
of P and T wave [7]. The scales 15 and 41 provide the greatest accuracy in detecting these waves. The wavelet "bior1.5" using scales 15 and 41 is shown in Figure 2.

![Wavelet 'bior1.5' Using Scales 15 and 41](image)

**Figure 2. Wavelet ‘bior1.5’ Using Scales 15 and 41**

3. **Methodology**

The process of analyzing the ECG signal can be divided into two stages: the stage of preprocessing and feature extraction.

Preprocessing means removing noise (electromyographycal potentials of muscles, artifacts of electrodes interaction with skin, amplifiers, electronic noise and background noise network) [3-5]. Noise is considered to be high-frequency cardio components. Noise removal leads to compression and smoothing the ECG signal.

The stage of feature extraction from cardio signal is the process of finding the required information (the teeth, complexes, etc.).

Let us consider each stage.

3.1. **Preprocessing**

In the simplest model the noisy signal is given by

\[
s(n) = f(n) + \sigma \cdot e(n)
\]

(2)

Where \( f(n) \) is a signal and \( \sigma \) is a noise level, \( e(n) \) is a Gaussian noise [4-5].

At wavelet analysis the signal is decomposed into approximate coefficients, which represent the smoothed signal, and the detailing coefficients that describe the vibration. Consequently, the noise component is better reflected in the detailing coefficients. Such components can be removed using the procedure of zeroing or recalculation of detail coefficients, whose values are less than the value threshold. Thresholding today is a perspective tool for the treatment of cardio noise (high frequency components).

**Types** of thresholding include

- Hard thresholding
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\[ f(x) = \begin{cases} x, & x \geq t, \\ 0, & x < t \end{cases} \] (3)

Where \( t \) is a thresholding.
- Soft thresholding
  \[ f(x) = \begin{cases} x-t, & x \geq t, \\ 0, & x < t, \\ x+t, & x \leq t \end{cases} \] (4)

Views of thresholding include
- Global thresholding;
- Local thresholding (general and multilevel).

Methods for determining the threshold include
- **SQR-LOG** method [12-13] by
  \[ t = \sqrt{\frac{2 \text{median}(\{c(i)\}, i = 1 \ldots n)^2}{0.6745}} \ln(n), \] (5)
  Where 0.6745 is a value of Gaussian noise, \( c(i) \) are wavelet coefficients;
- **Berg-Massar** method [14] by
  \[ z = \arg \min \left\{ - \sum \{c^2(i), i < k \} + 2\sigma^2 k \left( a + \ln \left( \frac{n}{k} \right) \right) \right\}, \] (6)
  \[ k = 1 \ldots n \]
  Where \( \sigma^2 \) is a variance, \( a \) is a special parameter (given by the researcher);
  The following ranges of parameters are
  - High (2.5 \( \leq a \leq 10 \) );
  - Middle (1.5 \( \leq a < 2.5 \) );
  - Low (1 \( \leq a < 1.5 \)).
- **Stein** method [15] by
  \[ T_m = \arg \min_{t \geq 0} [SURE(W)], \]
  \[ SURE(W) = \sigma^2 - \frac{1}{N} (2\sigma^2 \# \{ n \mid W(m,n) \} - \sum_{k=1}^{L} \min \{ W(m,n) \}^2) \] (7)

Daubechies wavelet was used as a base [16]. Limitation is the value of high and low frequency filters, which must be less than 10. Wavelets "db2" and the "db4" satisfy these requirements.

The upper frequency of the ECG signal which affects its shape should not exceed 100 Hz [4]. Therefore, the components of the signal frequencies above 100 Hz can be removed without significant changes in the waveform. Hence, we calculate the level of signal decomposition for wavelet “db2” and the “db4”. Wavelet “db2” has a center frequency \( Fr = 0.6667 \) Hz [4]. As \( \Delta t = 1/1024 \) then center frequency of the first level of decomposition will be \( Fr_1 = 0.6667 \times 1024 = 682.70 \) Hz, the second level will be \( Fr_2 = 341.35 \) Hz, the third level will be \( Fr_3 = 170.68 \), the fourth level will be \( Fr_4 = 85.34 \) Hz. Similarly, for the wavelet “db4” with a center frequency \( Fr = 0.7143 \) Hz [15]: \( Fr_1 = 734.30 \) Hz, \( Fr_2 = 367.15 \) Hz, \( Fr_3 = 183.57 \) Hz, \( Fr_4 = 91.8 \) Hz. Thus to remove the cardio component
frequencies above 100 Hz the use of fourth level of decomposition is required. We get a signal compression factor of 16.

While choosing the of wavelet basis we use the mean square error by

$$MSE(w,l) = \frac{1}{N} \sum_{i=1}^{N} (s(i) - \theta(i))^2,$$

(8)

Where $s(t)$ is a original signal, $\theta(i)$ is a filtered signal, $w$ is a wavelet function, $l$ is a decomposition level [16].

The results of wavelets “db2” and the “db4” at the fourth level of decomposition with the usage of local multilevel thresholding are presented in Tables 3-4.

**Table 3. Thresholding Results for Wavelet “db2”**

<table>
<thead>
<tr>
<th>Method</th>
<th>Hard thresholding</th>
<th>Soft thresholding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berg-Massar method</td>
<td>0.0447</td>
<td>0.0563</td>
</tr>
<tr>
<td>SQR-LOG method</td>
<td>2.5625</td>
<td>9.1393</td>
</tr>
<tr>
<td>Stein method</td>
<td>1.0046</td>
<td>1.0239</td>
</tr>
</tbody>
</table>

**Table 4. Thresholding Results for Wavelet “db4”**

<table>
<thead>
<tr>
<th>Method</th>
<th>Hard thresholding</th>
<th>Soft thresholding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berg-Massar method</td>
<td>0.0393</td>
<td>0.0530</td>
</tr>
<tr>
<td>SQR-LOG method</td>
<td>2.4873</td>
<td>8.9010</td>
</tr>
<tr>
<td>Stein method</td>
<td>1.0073</td>
<td>1.0309</td>
</tr>
</tbody>
</table>

The above results showed that the best wavelet support is the wavelet “db4” at the fourth level of decomposition, and the best method for a local multilevel hard wavelet thresholding is Berg-Massar method (Fig. 3-4).

![Figure 3. Original Signal](image-url)
3.2. Allocation of P-QRS-T complexes

Choosing the coordinates of important points of the ECG signal (onset, peak and offset T wave, QRS complex and P wave) in continuous wavelet transform requires the use of wavelet “bior1.5” with scale 15 for the detection of QRS-complex or scale 41 for the detection of P and T wave. In [7-11] the following approach is proposed: two neighboring zero crossing pairs of coefficients are chosen and between each of them is a local maximum (minimum), respectively; as a result, the first wavelet coefficient will be the beginning of the wave, the middle coefficient will be the wave peak, the last coefficient will be wave offset. This approach is applied to each of the scales.

The proposed method has a low speed (used in long-term monitoring) because of the constant search for a local maximum (minimum) in pairs of wavelet coefficients. Therefore, instead of searching for extremes, the authors propose to use the threshold value which is half of the global maximum (minimum). But the method does not provide correct detection systems if there is a significant amplitude difference between the QRS-complex and P and T wave. In this case, the method is ineffective. Therefore, for correct detection of P and T wave, the authors proposed to “delete” them (QRS-complexes) linearly approximating each part of this signal.

Thus the detection of important coordinates of the points of ECG signal consists of the following steps:
- Detection of QRS complexes.
  - Apply CWT with the scale factor which is equal to 15;
  - Calculate the thresholds \( t_1 = 0.5 \max |C(i)| \) and \( t_2 = 0.5 \min |C(i)| \);
  - Find zero crossing pairs coefficients;
Choose two neighboring zero crossing pairs of coefficients with a threshold \((t_1\) and \(t_2\)) between each of them. As a result, the first wavelet coefficient will be the beginning of the QRS complex, the middle coefficient will be the complex peak, the last coefficient will be complex offset.

- Apply the linear approximation of the signal parts between the onset and the offset of QRS-complexes.
- Detection of P and T waves.
  - Apply CWT with the scale factor which is equal to 41;
  - Calculate the thresholds \(t_1 = 0.5 \max(C(i)\) and \(t_2 = 0.5 \min(C(i)\);
  - Find zero crossing pairs coefficients;
  - Choose two neighboring zero crossing pairs of coefficients with a threshold \((t_1\) and \(t_2\)) between each of them. As a result, the first wavelet coefficient will be the beginning of the P wave, the middle coefficient will be the wave peak, the last coefficient will be wave offset;
  - Repeat the previous step for the T complex.

Stages of the procedure are illustrated in Fig. 5-9.

**Figure 5. Application of CWT with the Scale Factor which is Equal to 15 of the ECG Signal**

**Figure 6. Detecting QRS Complexes of ECG Signal**
Figure 7. “Deleting” QRS-Complexes from the ECG Signal

Figure 8. Application of CWT with Scale Factor which Equal to is 41 of the ECG Signal

Figure 9. Detecting P and T Waves of ECG Signal
4. Conclusions

- The choice of the type of wavelet transformation and the wavelet basis for analysis of ECG signal (CWT using a basis of “bior1.5”) has been found.
- The choice of the scale factor for the continuous wavelet transformation with the aim to detect P, QRS and T wave (scale 15 for the detection of QRS-complex and scale 41 for the detection of P and T waves has been made).
- Methods of P-QRS-T peaks detection have been improved as well as the threshold value and use of the approximation signal in QRS-complex in order to improve the accuracy of detection of P and T waves.
- Methods of signals refinement from noise have been analysed.
- The choice has been made of the type and method of wavelet thresholding (local multilevel hard thresholding using the Berg-Massar method and “db4” at the fourth level of decomposition as a basis).

The proposed method of the analysis of an ECG signal based on wavelet transformation in high-resolution electrocardiography system allows filtering the ECG signals without the loss of information.

References
