A Numerical Study on an Ammensal - Enemy Species Pair with Unlimited Resources and Mortality Rate for Enemy Species

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Abstract

The present paper spirits to study the interactions between the species in a mathematical model of an Ammensal - enemy species pair with unlimited resources in which the mortality rate of enemy species is greater than its birth rate. The model is built by a coupled system of first order non-linear ordinary differential equations. The only one equilibrium point is acknowledged and the criteria for its stability are deduced. The numerical solutions for this model are calculated for tracing the nature and the interactions between the species by using Runge-Kutta method of fourth order.

Keywords: *Equilibrium points, Normal steady state, stability, R.K method of fourth order.*

1. Introduction

Biological interactions have the remarkable significance in ecosystem. The organisms interact with each other to maintain the ecological balances in natural world. An organism's interactions with its environment are fundamental to the survival of that organism and the functioning of the ecosystem as a whole. Sign-mediated interactions in which molecules serve as signs are the characteristic feature of communicative interactions. Mathematical ecology was initiated by Lotka [10] and Meyer [11] followed by several mathematicians and ecologists. They contributed their might to the growth of this area of knowledge as reported in the treatises of Kapur [7, 8]. Srinivas. N.C. [12] studied competitive eco-systems of two and three species with limited and unlimited resources. Later, Lakshminarayan and Pattabhi Ramacharyulu [9] studied Prey-predator ecological models with a partial cover for the prey and alternate food for the predator. Acharyulu [1-6] and Pattabhi Ramacharyulu investigated some results on the stability of an enemy and Ammensal species pair with various resources.

The present paper deals with the formation and analytical/numerical study on a mathematical model of an Ammensal - enemy species pair with unlimited resources in which the mortality rate is considered for the enemy species. Runge-Kutta method of Fourth order and genetic algorithm are utilized to compute the numerical solutions of this model.

1.1 Notation adopted:

N_1, N_2 : The populations of the Ammensal (S_1) and enemy (S_2) species respectively at time t.
a_1, a_2 : The natural growth rates of S_1 and S_2
a_{12} : The Ammensal coefficient.

Further both the variables N_1 and N_2 are non-negative and the model parameters a_1, a_2, and a_{12} are assumed to be non-negative constants. Employing the above terminology, the model equations for a two species Ammensal system are constructed as below.

2. Basic Equations

The equation for the growth rate of the Ammensal species **(**S_1**)** is given

$$\frac{dN_1}{dt} = a_1 N_1 - a_{12} N_1 N_2 \tag{1}$$

The equation for the growth rate of the enemy species **(**S_2**)** is given

$$\frac{dN_2}{dt} = -a_2 N_2 \tag{2}$$

The system under the investigation has one equilibrium state given by

$$\overline{N_1} = 0; \ \overline{N_2} = 0$$

This is a state where both the species are washed out.
After linearization, we obtain

$\frac{dU_1}{dt} ?\ a_1U_1$ and $\frac{dU_2}{dt} ?$ 뺐 $_2U_2$ (3)

The characteristic equation is $(\lambda - a_1)(\lambda + a_2) = 0$

One root of this equation is $\lambda_1 = a_1$ which is positive and the other is $\lambda_2 = -a_2$ which is negative. Hence the equilibrium state is **unstable.**

The solutions of equations (3) are

$$U1 = U10\ e^{a_1 t}, \ U2 = U20\ e^{-a_2 t} \tag{4}$$

where U_{10}, U_{20} are the initial values of U_1 and U_2.

The solution curves are illustrated in Figure(1) and Figure(2)

Case 1 : $\mathbf{U_{10} < U_{20}}$ i.e initially the enemy species dominates the Ammensal species.

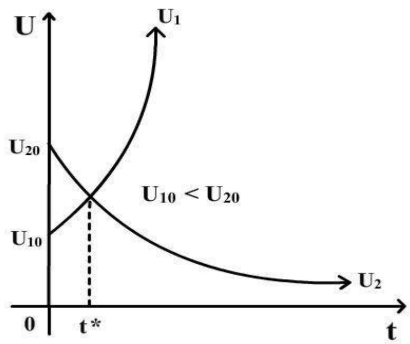


Figure 1

We notice that the enemy (S_2) while declining, dominates over the Ammensal(S_1) species up to the time -instant t*= $\frac{1}{(a_1 + a_2)}\log\left(\frac{U_{20}}{U_{10}}\right)$ there after the Ammensal(S_1)species dominates over the enemy (S_2) and the enemy (S_2) species declines further as shown in Figure.1

Case 2 : $U_{10} > U_{20}$ i.e. initially the Ammensal species dominates the enemy species.

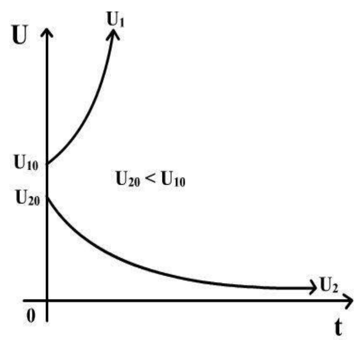


Figure 2

We notice that the Ammensal(S_1)species is going away from the equilibrium point while the enemy(S_2)species declines all throughout and approaches asymptotically to the equilibrium point as shown in Figure(2). Hence, the state is **unstable**.

2.1 Trajectories of Perturbed Species

The trajectories in $U_1 - U_2$ plane are given by

$$x^{a_2} y^{a_1} = 1 \tag{5}$$

where $x = \frac{U_1}{U_{10}}$, $y = \frac{U_2}{U_{20}}$ and are shown in Figure(3)

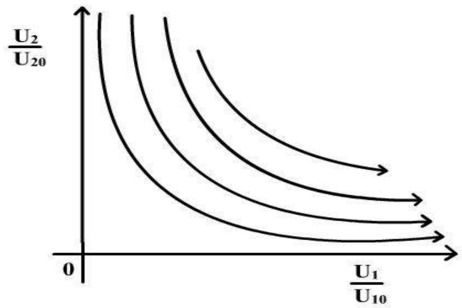


Figure 3

3. The Solutions of the Model are Computed by the Classical Runge-Kutta Method of Fourth Order with the help of Genetic Algorithm

The obtaining cases in this model are divided in to two cases.

Case(A): $a_1 > a_2$ and $N_{10} > N_{20}$
Case(B): $a_1 > a_2$ and $N_{10} < N_{20}$

We have found out all most all possible solutions in the specified interval by utilizing the classical RK method of fourth order with the help of Genetic Algorithm. The interval is conceived to range over from 0 to 5 for noting the nature of the model. The achieved solutions are arranged in the tabular forms and the graphs are exemplified wherever essential.

3.1.Case(A): The initial strength of the Ammensal species is greater than the initial strength of enemy species in which the natural growth rate of Ammensal species is greater than the natural growth rate of enemy species in which the i.e $a_1 > a_2$ and $N_{10} > N_{20}$ .The incurred solutions are established in the Table-1.

Table-1

S.N0	a_1	a_{12}	a_2	N_1	N_2	t
1	2.63732	4.373033	0.820299	1.44198	1.388942	0
2	3.72288	3.216121	2.408229	2.829155	2.580276	0.22
3	1.959451	1.387418	0.429724	4.802141	2.859251	0.4
4	4.056076	4.703185	2.567189	2.639085	2.431146	0.46
5	2.939548	4.619774	0.91399	3.808883	1.521413	0.48
6	4.161373	4.927467	3.003534	4.00152	3.024302	0.51
7	3.857824	1.638284	0.768296	4.624799	3.803581	0.65
8	4.04927	4.743952	3.252085	4.879934	3.878433	0.66
9	4.363967	2.023918	0.768296	4.994785	3.803581	0.99
10	1.959451	1.387418	0.606044	4.39576	3.849325	2.73
11	1.959451	1.387418	0.416091	4.802141	3.477042	3.7
12	1.959451	1.387418	0.544645	4.802141	4.431059	3.95
13	1.959451	1.387418	0.484915	4.802141	4.668355	4.96
14	1.959451	1.387418	0.483626	4.802141	4.668355	4.98

Some of the recognized solution curves are illustrated from Figure(4) to Figure(10)

Figure (4) ; S.N0-1 in Table-1

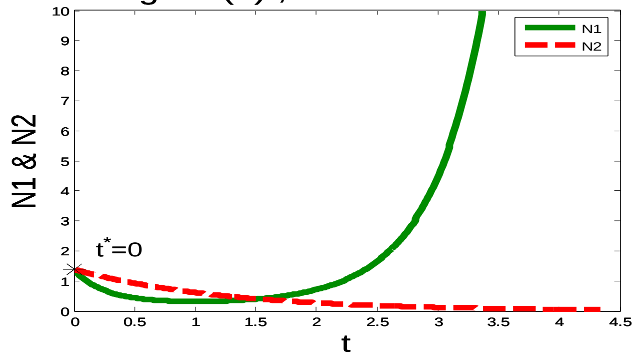


Figure (5) ; S.N0-3 in Table-1

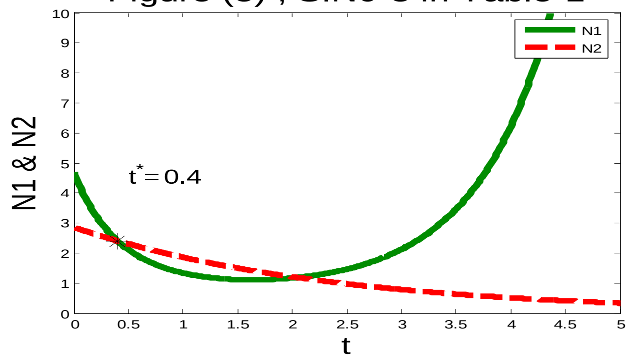


Figure (6) ; S.N0-5 in Table-1

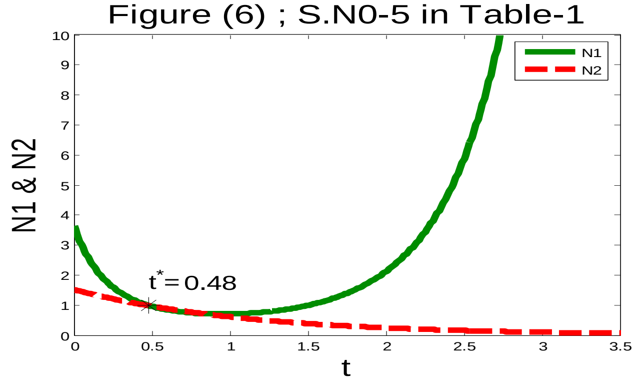
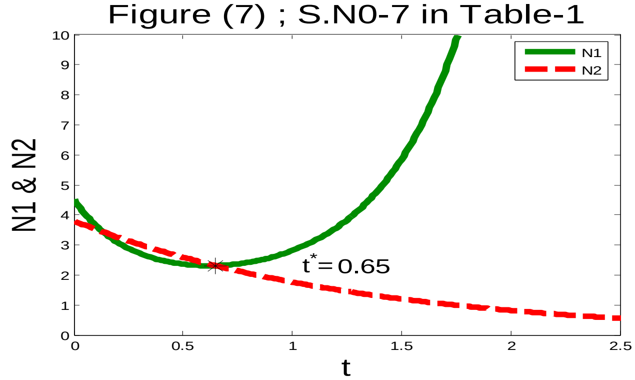
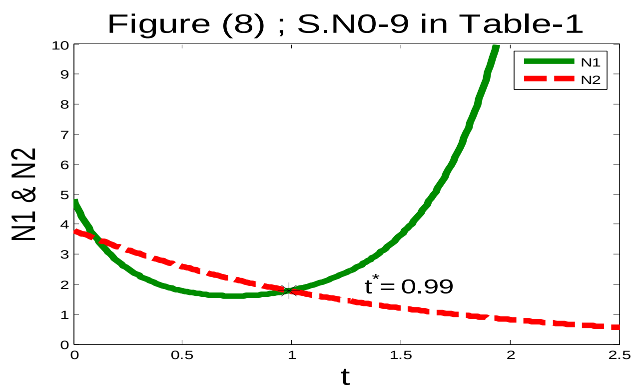
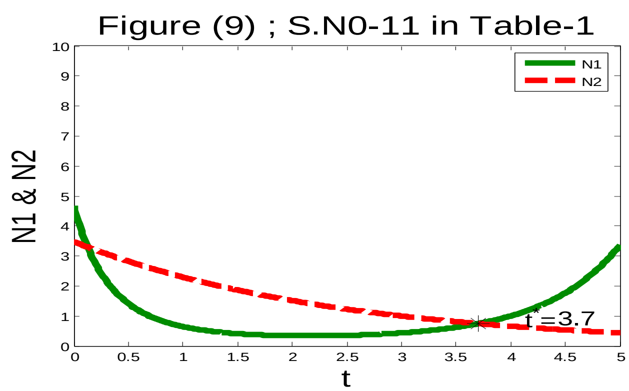
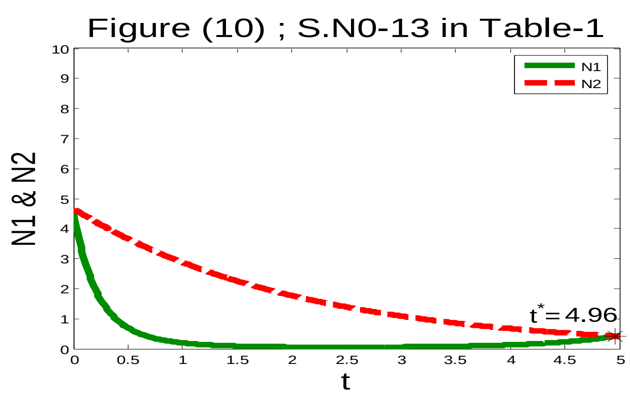


Figure (7) ; S.N0-7 in Table-1









3.2. Conclusions:

(i) In the initial stage Ammensal (S_1) species masters over Enemy (S_2) species in natural growth as well as in its initial population.

(ii) The Enemy species masters the Ammensal until the dominance- reversal time t* after that the Ammensal outnumbers the Enemy in the specified interval in the second phase.

(iii) The enemy (S_2) species declines all throughout and approaches asymptotically to the equilibrium point

(iii) Even though Ammensal blooms in growth rate after t*, it gradually decays in the remainig interval.

4.0 Case(B): The initial strength of the Ammensal species is less than the initial strength of enemy species in which the natural growth rate of Ammensal species is greater than the natural growth rate of enemy species. i.e $a_1 > a_2$ and $N_{10} < N_{20}$ .. The obtained solutions are given in the Table-2.We have traced out all most all possible solutions in the finite interval by employing the classical RK method of fourth order with the aid of Genetic Algorithm. The interval is assumed to range over 0 to 5 for observing the nature of the model. The gained solutions are framed in the tabular form and the graphs are illustrated wherever necessary.

Table-2

S.N0	a1	a12	a2	N1	N2	t
1	4.274589	0.806386	2.68793	1.675777	2.46473	0.07
2	1.91035	0.409059	0.869045	2.874402	3.568079	0.15
3	4.015736	2.23371	2.911426	3.714524	3.781251	0.17
4	3.857689	3.157831	3.070795	2.388588	2.580276	0.17
5	4.904487	0.291437	1.795517	0.494222	2.264397	0.24
6	1.984803	1.544639	0.820299	1.004814	1.396502	0.35
7	2.63732	2.057763	0.819621	0.992496	1.405051	0.36
8	4.904487	0.291437	1.483514	0.537283	4.745819	0.4
9	4.904487	0.291437	1.483514	0.537283	4.745819	0.4
10	3.945202	3.495429	3.771943	0.917421	4.297107	0.68
11	4.123177	4.880109	2.717898	0.752771	2.421957	0.72
12	4.639391	4.743952	3.252085	1.902184	3.878433	0.75
13	2.63732	3.235634	0.820299	1.004814	1.388942	0.95
14	1.769472	0.039136	0.048019	0.397832	2.253894	0.99
15	1.583106	0.223243	0.143179	0.8384	2.703017	1
16	2.843387	4.456947	2.81503	1.585151	3.710978	1.15
17	4.325119	4.792021	2.512733	1.87465	4.416034	1.32
18	1.731647	0.164952	0.07173	0.397832	3.088219	1.54
19	1.769472	0.164952	0.048019	0.397832	3.11923	1.55
20	1.717915	0.164952	0.07173	0.397832	3.11923	1.57
21	1.717915	0.164952	0.07173	0.397832	3.11923	1.57
22	2.63732	4.68519	0.820299	1.004467	1.388942	1.9
23	4.750537	4.793718	0.986622	2.338072	4.010896	3.37
24	4.70023	4.793718	1.039342	1.903309	4.361788	3.56
25	4.70023	4.793718	1.032974	1.920234	4.361788	3.58
26	4.70023	4.793718	1.032974	1.920234	4.361788	3.58
27	4.342145	4.793718	1.032974	1.816732	4.361788	3.85
28	4.562472	2.420851	0.34923	2.278109	3.801376	4.26
29	4.562472	2.420851	0.34923	2.278109	3.801376	4.26
30	4.562472	2.420851	0.34923	2.278109	3.801376	4.26
31	2.290195	4.595444	0.820299	2.62861	2.647919	4.62
32	1.959451	1.387418	0.417441	1.166893	3.492113	4.64
33	2.187173	1.44759	0.447998	1.989651	4.130102	4.73
34	4.156118	2.420851	0.34923	2.278109	3.801376	4.91

Some of the picked out solution curves are depicted from Figure (11) to Figure (19)

Figure (11) ; S.N0-1 in Table-2

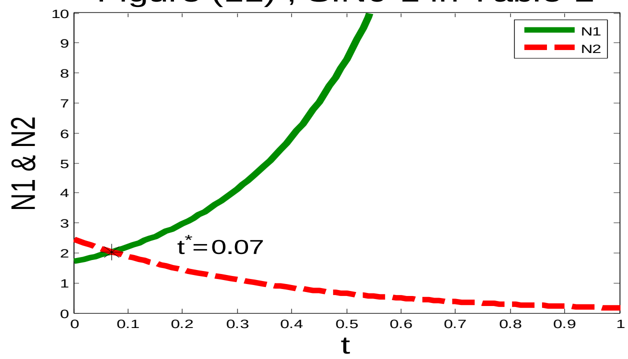


Figure (12) ; S.N0-4 in Table-2

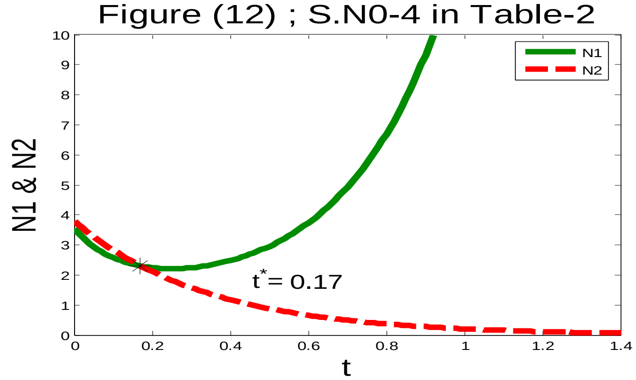


Figure (13) ; S.N0-8 in Table-2

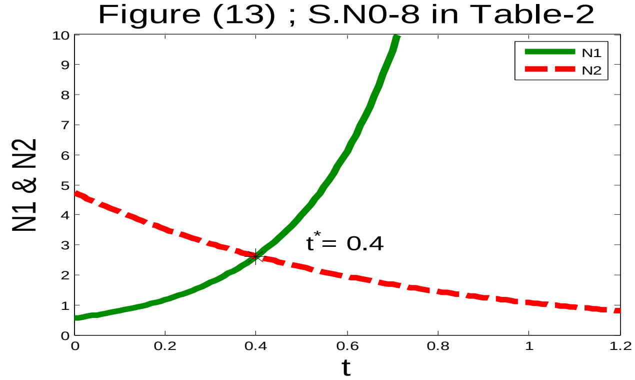


Figure (14) ; S.N0-12 in Table-2

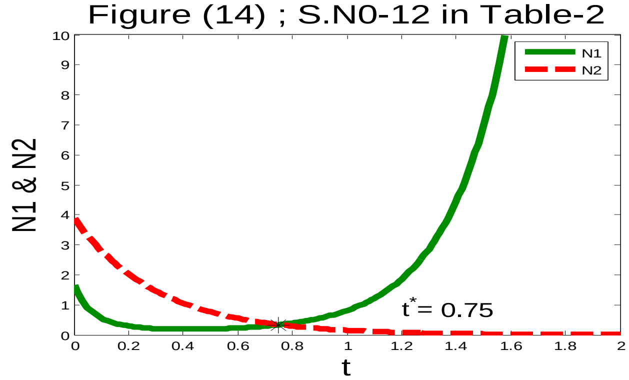


Figure (15) ; S.N0-12 in Table-2

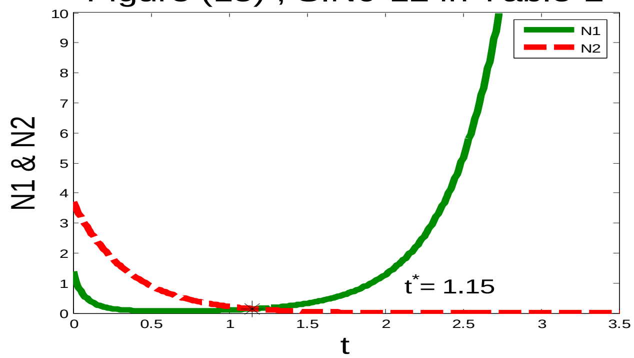


Figure (16) ; S.N0-20 in Table-2

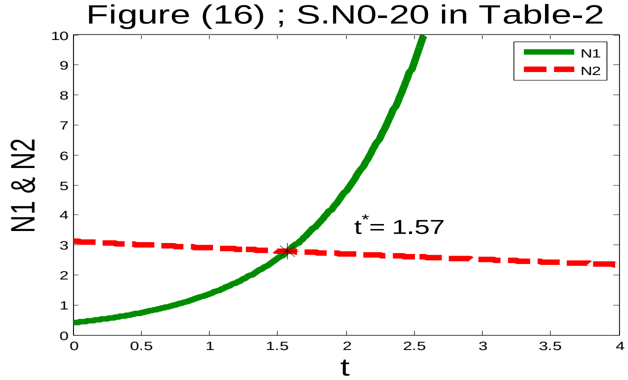


Figure (17) ; S.N0-24 in Table-2

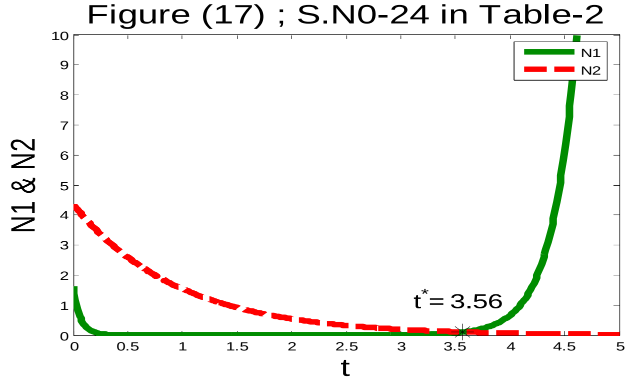
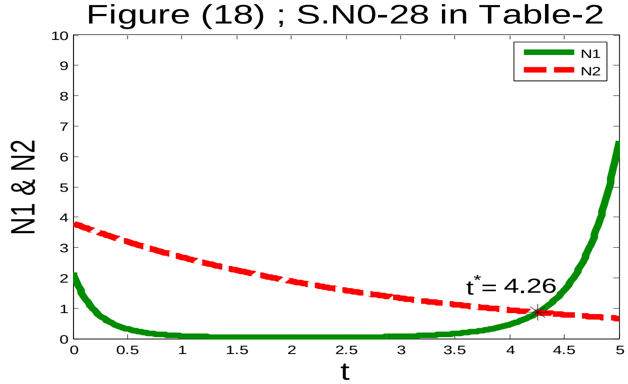
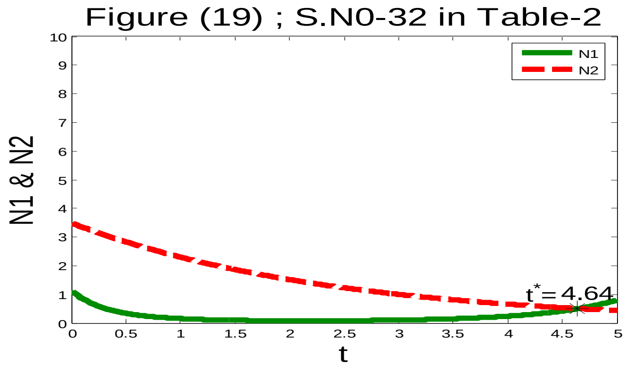


Figure (18) ; S.N0-28 in Table-2





4.1. Conclusions:

(i) It is evident that the enemy dominates over the Ammensal species in initial population strength where as Ammensal dominates enemy in natural growth rate.
(ii) The enemy species surpasses Ammensal species till the dominance- reversal time t* after that the Ammensal surpasses the enemy species in the taken interval. Further it is noticed that there is a steep down in the Ammensal spieces and the enemy diminishes gradually with exponential decay.
(iii) Both the species maintain steady variation with low growth rates at the end of the interval. More over these two species suffer a steep fall and appear to be almost extinct with negligible growth rates.

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