Feature-preserving smoothing of point-sampled geometry

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Abstract

Based on noise intensity, in this paper, we propose a feature-preserving smoothing algorithm for point-sampled geometry (PSG). The noise intensity of each sample point on PSG is first measured by using a combined criterion. Based on mean shift clustering, the PSG is then clustered in terms of the local geometry-features similarity. According to the cluster to which a sample point belongs, a moving least squares surface is constructed, and in combination with noise intensity, the PSG. is finally denoised. Some experimental results demonstrate that the algorithm is robust, and can smooth out the noise efficiently while preserving the surface features.

1. Introduction

Point-sampled geometry (PSG) without topological connectivity is normally generated by sampling the boundary surface of physical 3D objects with 3D-scanning devices. Despite the steady improvement in scanning accuracy, undesirable noise is inevitably introduced from various sources such as local measurements and algorithmic errors. Thus, noisy models need to be smoothed or denoised before performing any subsequent geometry processing such as simplification, reconstruction and parameterization. It remains a challenging task to remove the inevitable noise while preserving the underlying surface features in computer graphics.

Earlier methods such as Laplacian [1] for denoising PSG are isotropic, which result commonly in point drifting and oversmoothing. So the anisotropic methods were introduced. Clarenz et al. [2] presented a PDE-based surface fairing application within the framework of processing point-based surface via PDEs. Lange and Polthier [3] proposed a new method for anisotropic fairing of a point sampled surface based on the concept of anisotropic geometric mean curvature flow. Based on dynamic balanced flow, Xiao et al. [4] presented a novel approach for fairing PSG. Other methods have also been proposed for denoising the PSG. Algorithms that recently attracted the interest of many researchers are moving-least squares (MLS) approaches [5-7] to fit a point set with a local polynomial approximation; the point set surfaces can be smoothed by shifting point positions towards the corresponding MLS surfaces. The main problem of MLS-based methods is that prominent shape features are blurred while smoothing PSG.

Concerning the above problem of MLS approaches, this paper puts forward a smoothing algorithm for PSG based on noise intensity. We adopt the combined criterion presented by Weyrich et al in [6] to measure the noise intensity of each sample point. In order to take into account the similarity of geometry features while smoothing PSG, we introduce mean shift clustering method. In combination with noise intensity and the constructed MLS surfaces based on these clusters, we achieve fairing of PSG with feature preservation.

2. Measuring the noise intensity

Weyrich et al [6] proposed a combined criterion, which is calculated as a weighted combination of *plane fit criterion*, *miniball criterion* and *nearest-neighbor reciprocity criterion*, to estimate the likelihood for a sample point to be an outlier. We introduce this combined criterion and consider this likelihood as the *noise intensity* of sample point. We briefly review them as follows.

The *plane fit criterion* considers a plane H that minimizes the squared distances to $\boldsymbol{p}_i$'s k nearest neighbors $N_k(\boldsymbol{p}_i)$, i.e.,

$$\min_H \sum_{\boldsymbol{q} \in N_k(\boldsymbol{p}_i)} \text{dist}(\boldsymbol{q}, H)^2 .$$

The plane fit criterion is defined as $\chi_{\text{pl}}(\boldsymbol{p}_i) = d / (d + \bar{d})$, where d is the distance of $\boldsymbol{p}_i$ to H, and $\bar{d}$ the mean distance of points from $N_k(\boldsymbol{p}_i)$ to H.

For each point $\boldsymbol{p}_i$ consider the smallest enclosing sphere S around $N_k(\boldsymbol{p}_i)$. S can be seen as an approximation of the k-nearest-neighbor cluster. Comparing $\boldsymbol{p}_i$'s distance d to the center of S with the sphere's diameter ($2r$) yields a measure for $\boldsymbol{p}_i$'s likelihood to be an outlier. Consequently, the *miniball criterion* is defined as $\chi_{\text{mb}}(\boldsymbol{p}_i) = d / (d + 2r / \sqrt{k})$.

Observe a directed graph G of k-neighbor relationships: Outliers are assumed to have a high number of uni-directional exitant edges, i.e., asymmetric neighbor relationships. Consequently the *nearest-neighbor reciprocity criterion* considers the ratio between unidirectional and bi-directional exitant edges in G. The uni-directional neighbors are defined as

$$N_{\text{uni}}(\boldsymbol{p}_i) = \{\boldsymbol{q} \mid \boldsymbol{q} \in N_k(\boldsymbol{p}_i), \boldsymbol{p}_i \notin N_k(\boldsymbol{q})\},$$

while the bi-directional neighbors build a set

$$N_{\text{bi}}(\boldsymbol{p}_i) = \{\boldsymbol{q} \mid \boldsymbol{q} \in N_k(\boldsymbol{p}_i), \boldsymbol{p}_i \in N_k(\boldsymbol{q})\}$$

So the criterion is defined as

$$\chi_{\text{bi}}(\boldsymbol{p}_i) = \|N_{\text{uni}}(\boldsymbol{p}_i)\| / k$$

By combining the three criteria and using weights w_1, w_2 and w_3 ($w_1+w_2+w_3=1$), in this paper, we compute the noise intensity of δ_i sample point $\boldsymbol{p}_i$ as

$$\delta_i = w_1 \chi_{\text{pl}}(\boldsymbol{p}_i) + w_2 \chi_{\text{mb}}(\boldsymbol{p}_i) + w_3 \chi_{\text{bi}}(\boldsymbol{p}_i) \quad (1)$$

3. Mean shift clustering for PSG

By using mean shift clustering method, in this paper, the PSG is clustered so as to consider the similarity of geometry features while smoothing it. The mean shift algorithm is a nonparametric clustering technique for the analysis of a complex multimodal feature space and the delineation of arbitrarily shaped clusters [8], and it has a wide variety of applications in the fields of computer vision and pattern recognition. Recently it has been extended to the field of digital geometry processing [9,10]. In the following we first present a short review of the adaptive mean-shift technique and then describe how to apply it to PSG.

Assume that each data point $\boldsymbol{x}_i \in R^d$, $i = 1, \ldots, n$ is associated with a bandwidth value $h_i > 0$. The *sample point* estimator [9,11]

$$\hat{f}_K(\boldsymbol{x}) = \frac{1}{nh_i^d} \sum_{i=1}^{n} K\left(\left\| \frac{\boldsymbol{x}-\boldsymbol{x}_i}{h_i} \right\|^2\right) \tag{2}$$

based on a spherically symmetric kernel K with bounded support satisfying

$$K(\boldsymbol{x}) = c_{k,d}\, k(\|\boldsymbol{x}\|^2) > 0 \quad \|\boldsymbol{x}\| \le 1$$

is an adaptive nonparametric estimator of the density at location $\boldsymbol{x}$ in the feature space. The function $k(x),\ 0 \le x \le 1$, is called the *profile* of the kernel, and the normalization constant $c_{k,d}$ assures that $K(\boldsymbol{x})$ integrates to one. The function $g(x) = -k'(x)$ can always be defined when the derivative of the kernel profile $k(x)$ exists. Using $g(x)$ as the profile, the kernel $G(x)$ is defined as $G(x)=c_{g,d}\, g(\|x\|^2)$

By taking the gradient of equation (2) the following property can be proven

$$\boldsymbol{m}_G(\boldsymbol{x}) = \mathrm{C}\frac{\hat{\nabla} f_K(\boldsymbol{x})}{\hat{f}_G(\boldsymbol{x})}$$

where C is a positive constant and

$$\boldsymbol{m}_G(\boldsymbol{x}) = \frac{\sum_{i=1}^{n} \frac{\boldsymbol{x}_i}{h_i^{d+2}} g\left(\left\| \frac{\boldsymbol{x}-\boldsymbol{x}_i}{h_i} \right\|^2\right)}{\sum_{i=1}^{n} \frac{1}{h_i^{d+2}} g\left(\left\| \frac{\boldsymbol{x}-\boldsymbol{x}_i}{h_i} \right\|^2\right)} - \boldsymbol{x} \tag{3}$$

is called the *mean shift vector* pointing toward the direction of maximum increase in the density. A gradient-ascent process with an adaptive step size

$$\boldsymbol{y}^{[j+1]} = \boldsymbol{m}_G(\boldsymbol{y}^{[j]}), \quad j=0,1,2,... \tag{4}$$

constitutes the core of the mean shift clustering procedure. For clustering $S=\{\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_n\}$ with mean shift, the following two steps are performed on each $\boldsymbol{x}_i \in S$: (a) Initialize $\boldsymbol{y}_i^{[0]}$ with $\boldsymbol{x}_i$; (b) Compute $\boldsymbol{y}_i^{[j]}$ according to equation (4) until convergence. It can be shown in the literature [8] that under some general assumptions the sequences $\{\boldsymbol{y}_i^{[j]}\}$ converge to the points where $\hat{f}_K(\boldsymbol{x})$ defined by equation (2) attains its local maxima (mode). Accordingly, the points that converge to the same mode are associated with the same cluster.

One simple extension of the above clustering procedure consists of dealing with a set S, each element of which has two components of a different nature, $S=\{\boldsymbol{x}_i = (\boldsymbol{c}_i, \boldsymbol{q}_i) : \boldsymbol{c}_i \in C,\ \boldsymbol{q}_i \in Q\}$. In such a situation, it is convenient to use the mean shift clustering procedure with separable kernels

$$\hat{f}_K(\boldsymbol{x}) = \frac{1}{nh_{i_1}^{d_1} h_{i_2}^{d_2}} \sum_{i=1}^{n} k_1\left(\left\| \frac{\boldsymbol{c}-\boldsymbol{c}_i}{h_{i_1}} \right\|^2\right) k_2\left(\left\| \frac{\boldsymbol{q}-\boldsymbol{q}_i}{h_{i_2}} \right\|^2\right)$$

.

In this paper, we consider the sample points $\{\boldsymbol{p}_i\}$ equipped with the normals $\{\boldsymbol{n}_i\}$ and the mean curvature $\{H_i\}$ as scattered data $S=\{\boldsymbol{x}_i = (\boldsymbol{c}_i, \boldsymbol{q}_i) : \boldsymbol{c}_i = \boldsymbol{p}_i,\ \boldsymbol{q}_i = (\boldsymbol{n}_i, H_i)\}$ in R^7. For both $\boldsymbol{c}_i$ and $\boldsymbol{q}_i$, we use the normal kernel. For the bandwidth values h_i, there are numerous methods to define them, most of which use a pilot density estimate. The simplest way to obtain the pilot density estimate is by nearest neighbors. To accelerate the mean shift computation, we construct a *k*-D tree for the point set $\{\boldsymbol{c}_i\}$. According to the *k*-nearest neighbors $N_k(\boldsymbol{c}_i)$ of $\boldsymbol{c}_i$, we can adaptively take $h_{i_1} = \|\boldsymbol{c}_i - \boldsymbol{c}_{i,k}\|_2$, where $\boldsymbol{c}_{i,k}$ is the *k*-nearest neighbor of $\boldsymbol{c}_i$, and $h_{i_2} = max\{\|\boldsymbol{q}_i - \boldsymbol{q}_{i,1}\|_2, \ldots, \|\boldsymbol{q}_i - \boldsymbol{q}_{i,k}\|_2\}$. After clustering for PSG by using this mean shift technique, the geometry features of the points in the same cluster, which contain the point positions, normals and mean curvatures, are locally similar, respectively. Fig.1c demonstrates the mean shift clustering of the Face model, and its point set of local modes is illustrated in Fig.1d.

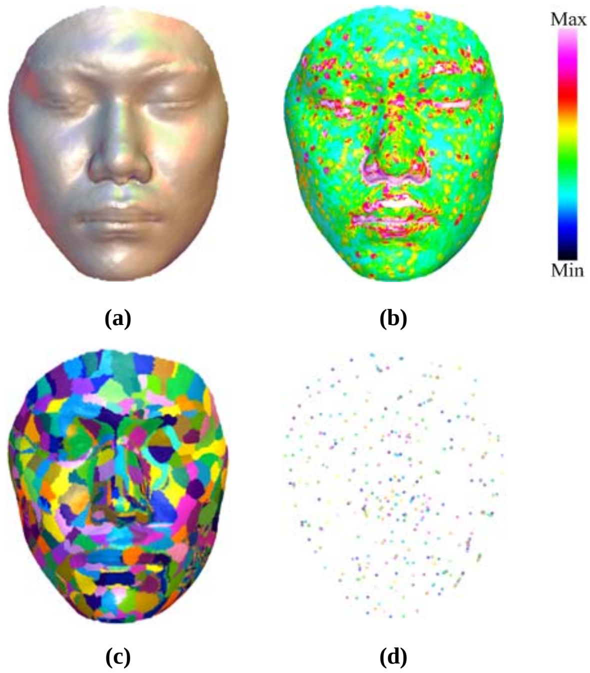

Fig.1 (a) Noisy Face model; (b) Noisy model colored by mean curvature; (c) Our mean shift clustering; (d) The point set of local modes.

4. Smoothing of PSG

The main idea of smoothing algorithm is as follows: MLS surfaces are first constructed in terms of the above clusters, and in combination with the noise intensity, the offset distances of sample points are computed. According to offset distances, we shift sample point positions along their own normals to eliminate the noise from PSG.

Alexa et al.[12] proposed a PSG representation by fitting a local polynomial approximation to the point set using a MLS method. The result of the MLS-fitting is a smooth, 2-manifold surface for any point set. Given a point set $P = \{\boldsymbol{p}_i\}$, the continuous MLS surface *S* is defined implicitly as the stationary set of a projection operator $\psi(\boldsymbol{r})$ that projects a point onto the MLS surface. To evaluate ψ, a local reference plane $H = \{\boldsymbol{x} \in \mathbb{R}^3 | \boldsymbol{n} \cdot \boldsymbol{x} - D = 0\}$ is first computed by minimizing the weighted sum of squared distances, i.e.,

$$\arg\min_{\boldsymbol{n},\boldsymbol{q}} \sum_{\boldsymbol{p}_i \in P} (\boldsymbol{n}\cdot\boldsymbol{p}_i - \boldsymbol{n}\cdot\boldsymbol{q})^2 \theta(\|\boldsymbol{p}_i - \boldsymbol{q}\|)$$

where $\boldsymbol{q}$ is the projection of $\boldsymbol{r}$ onto H and θ is the MLS kernel function $\theta(d) = \exp(-d^2/h^2)$, where h is a global scale factor. Then a bivariate polynomial $g(u,v)$ is fitted to the points projected onto the reference plane H using a similar weighted least squares optimization. The projection of $\boldsymbol{r}$ onto S is given as $\psi(\boldsymbol{r}) = \boldsymbol{q} + g(0,0)\cdot\boldsymbol{n}$. More details on the MLS method can be found in [12].

Let $\boldsymbol{q}_i$ be the projection of $\boldsymbol{p}_i$ onto the corresponding MLS surface approximating the cluster of $N_k(\boldsymbol{p}_i)$. Although the noise from PSG is eliminated by shifting $\boldsymbol{p}_i$ to $\boldsymbol{q}_i$, the underlying surface features are blurred and the cluster of $N_k(\boldsymbol{p}_i)$ takes only into account the position relationship among sample points, not the similarity of geometry features.

We employ MLS surfaces to approximate the clusters of PSG obtained by using mean shift clustering method, and based on the noise intensity, the offset distance d_i of each sample point $\boldsymbol{p}_i$ is determined. Accordingly, the smoothed position $\boldsymbol{p}_i^*$ is given by $\boldsymbol{p}_i^* = \boldsymbol{p}_i + \lambda d_i \boldsymbol{n}_i$, where $\lambda (0 \le \lambda \le 1)$ is a user-adjustable smoothing parameter; d_i is expressed as $d_i = w(\delta_i)\cdot(-1)^\tau \cdot \| \boldsymbol{p}_i \boldsymbol{q}_i \|$, where the term $w(x)$=exp(-exp(-x)) denotes that the influence on the offset distance d_i increases with an increase in the noise intensity so that the regions with high noise intensity can be efficiently smoothed, and τ is set to 0 when $\boldsymbol{n}_i \cdot \boldsymbol{p}_i \boldsymbol{q}_i > 0$, which indicates that $\boldsymbol{p}_i$ is move along $\boldsymbol{n}_i$; otherwise τ is set to 1 and $\boldsymbol{p}_i$ is move along $-\boldsymbol{n}_i$.

5. Experimental results and discussion

In our experiments, we use Microsoft Visual C++ programming language on a personal computer with a Pentium IV 2.8 GHz CPU and 1 GB main memory. We have implemented our noise intensity-based smoothing (NIB) and another two denoising techniques: the Bilateral denoising (BIL) and the MLS-based denoising to compare their denoising results. We demonstrate two models in our comparison: a noisy Face model with 34 308 sample points (Fig.1a) and a noisy Armadillo-leg model with 93397 sample points (Fig.3a).We use the visualization scheme of mean curvature to compare this two techniques with our method; all the models are rendered by using a point-based rendering technique..

In Fig.2, we demonstrate a comparison of the denoised Face models by MLS, BIL and NIB. The denoised models are illustrated in the top row of Fig.2, and their corresponding mean curvature visualizations in the bottom row. As seen in Fig.2, our NIB removes the high-frequency noise properly and achieves a more accurate result than MLS or BIL does. Fig.3 shows a comparison of MLS, BIL and NIB concerning feature preservation. Note that our NIB preserves sharp features more accurately than MLS or BIL does while producing a smooth result.

Due to take not only into account the noise intensities of sample points but also the similarity of geometry features while smoothing PSG, our algorithm can remove the high-frequency noise properly and achieve a more accurate denoising result than BIL or MLS.

6. Conclusion

Based on noise intensity, in this paper, we presented a smoothing algorithm for PSG. By using mean shift clustering method, the PSG is clustered into clusters according to the similarity of geometry features. According to these clusters, the corresponding MLS surfaces are constructed and in terms of the noise intensity, the offset distance of each sample point is determined. In combination with the MLS surfaces and offset distances, the PSG is smoothed.

Our experimental results demonstrate that the proposed algorithm is robust, and can smooth out the noise efficiently while preserving the surface features.

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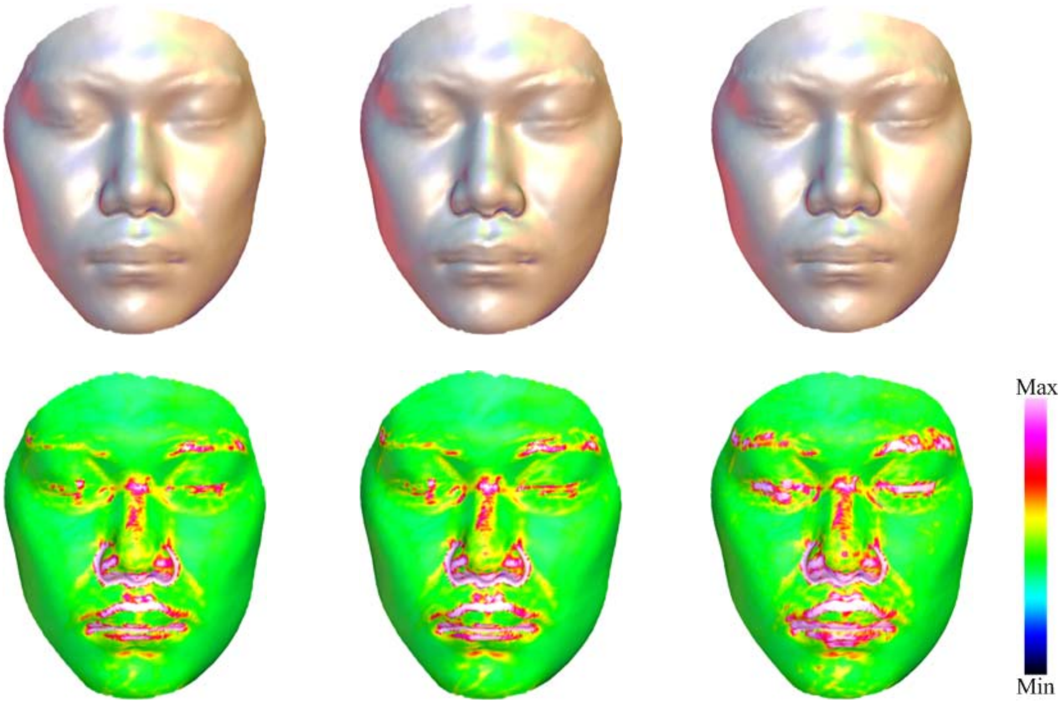

(a) MLS **(b) BIL** **(c) NIB**

Figure 2: Denoising noisy Face model (Fig.1a). Top: the denoised models. Bottom: the corresponding denoised model colored by mean curvature. Mean curvature coloring helps us to compare their corresponding fine details.

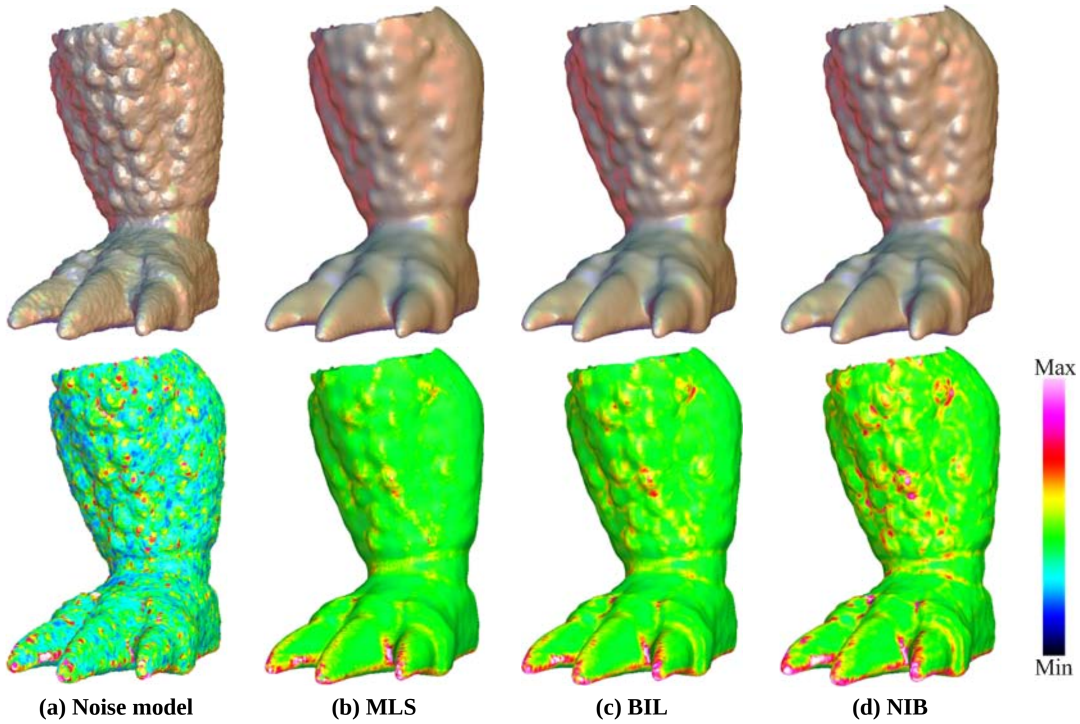

Fig.3 Denoising the noisy Armadillo-leg model. Top: the denoised models. Bottom: the corresponding denoised model colored by mean curvature.

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