A Multi Objective Approach to solve Capacitated Vehicle Routing Problems with Time Windows Using Mixed Integer Linear Programming

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Abstract

This paper presents a multi objective approach to solve a Capacitated Vehicle Routing Problem with Time Windows (CVRPTW). The proposed model was implemented and tested in a real life problem of a distribution company "Just in Time Delivery S.A" in Portugal. In this paper we have considered an objective function with two main goals: the first is to minimize the total number of vehicles used in the distribution of the commodities to the several clients and the second is to minimize the travelling time of the used vehicles. The proposed model has been solved numerically using the GLPK software and the optimal solution is presented.

Keywords: *CVRPTW, CVRP, VRP, Multi objective, GLPK, Mixed integer, Optimal solution.*

1. Introduction

In recent years, the researches in Optimization and its applications in diverse areas have been increased surprisingly. The optimal strategies are the key concerns in all aspects of engineering, commerce and biological sciences. See for examples [1] and [2] for the detail studies on applications of optimal control theory in Optimization. However, the optimal distribution management of the products is the main issue to any industry as well as to manufacturing company in the present age of competitive global economy. This is why from the last few decades the supply chain management is paid a remarkable attention to every large enterprise and/or manufacturing company. The Enterprise reduces their transporting and distributing cost through the manner of subcontractors and shared transportation. The customer request for sending time of the goods is getting more strictly; as the living standards have increased, customers pay increasing attention to the accuracy of product delivery times. As a result, the planning of delivery routes must consider customers' acceptable service time window.

Therefore, how to reduce the cost while delivering on time, namely, vehicle routing problems with time windows (VRPTW), has become an important issue in supply chain management [5].

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The vehicle routing problem (VRP) is one of the most challenging areas of research in the field of Combinatorial Optimization. Defined more than 40 years ago, this problem consists in designing the optimal set of routes for fleet of vehicles in order to serve a given set of customers. The interest in VRP is motivated by its practical relevance as well as by its considerable difficulty. Vehicle routing problem is a generic name given to a whole class of problems in which a set of routes for a fleet of vehicles based at one or several depots must be determined for a number of geographically dispersed cities or customers. This is a classic problem of operations research.

The most elementary version of the vehicle routing problem is the capacitated vehicle routing problem (CVRP). The capacitated vehicle routing problem (CVRP) is a generalization of VRP, first proposed by Dantzig and Ramser in 1959. CVRP is an important problem in the fields of transportation, distribution and logistics [7] which involves in finding a set of routes, starting and ending at a depot, that together cover a set of customers [14]. Till to date many models have been proposed where most of the proposed algorithms assume that the number of vehicles is unlimited and the objective is to obtain a solution that either minimizes the number of vehicles and/or total travel cost. However, transport operators in the real world face resource constraints such as a fixed fleet. For example, if the given problem is over-constrained in the sense of insufficient vehicles, but optimal strategy will be carried out so as to satisfy the demands of the clients within a time constraint. We propose such a model where vehicle is also limited due to the optimal resources management. In this case, ccustomers' demand must be satisfied by exactly one vehicle within one route, while respecting the vehicle's capacity [3]. In this work it is considered the CVRP with time windows (CVRPTW) that is one of the varieties of VRP [15]. This means that the vehicles must arrive to the clients in a limited time window fixed by them. The objective of CVRPTW is to design the shortest path for minimum traveling costs and number of vehicles without violating the constraints of time windows and loading capacity of vehicle. It is of great interest because of its practical relevance in real life. We propose here the optimal distribution management for a particular case implemented on a manufacturing and distribution company named "Just in Time Delivery S.A." in Portugal. For a mixed integer type linear programming problem our model provided good results.

The Just in Time Delivery S.A. is a distribution company that has to schedule the route of a fleet of vehicles which number is to be minimized. Their clients have a restricted time window to accept the delivery of the goods by the vehicles drivers, and thus they need to be there within that time window. The Just in Time Delivery S.A. also wants some sensibility analyzes, namely the capacity of the vehicles and a change of the time window of one of the clients.

There are different techniques proposed by several authors over the years that can be found in the literatures (see for examples [3], [5] and [6]) to deal with those problems, but due to the combinatorial dimension of the problem, heuristics and meta-heuristics are the largely used, because of their characteristics in dealing with this kind of issues. See for examples [8], [13], [11] and [12] where some more techniques of using Meta Heuristics and Heuristics can be found. In our study, we tested on a specific problem which is a small one and in this case a mixed integer type linear programming solver is used to solve the problem. However, mixed inter linear programming is a method that can deal only with small problems in particular territory, but performs good result. The main goal of this work is to find a feasible solution to the CVRPTW problem with 6 delivery points (clients), considering in this problem the following characteristics:

- one depot with non-limited capacity of one single commodity;

- a fleet of vehicles, with limited and known capacity, which is necessary to minimize in number;
- a set of 6 clients with service time window, demand of the commodity, distance time matrix;

Let us assume that travel time is proportional to distance, meaning that the vehicle runs at constant average velocity.

such that:

- each client is served within his time window by exactly one vehicle;
- the capacity of the vehicles is not exceeded;
- minimizing the number of vehicles that are used and the sum of the total travelling time.

2. Mathematical Analysis and Problem Formulation

The main objective of this work can be stated as follows: find a feasible solution to the schedule of the vehicles in a set of routes, so as to minimize both, the number of vehicles used and the total travel time.

The problem is formulated by a Graph *G (N,T)* where N is the number of clients plus the depot and T the network connections between the clients and the depot [12]. The connections are given by the travel time. We consider a fleet of identical vehicles {1, 2, ...v}, a set of clients {1, 2, ...*i*}, one depot represented by node 0 and n+1 as in [12]. All the vehicles have to depart from node 0 and arrive at the node n+1. If any vehicle departs from 0 and arrive at n+1 without passing any client (an empty route), it means that this vehicle is not necessary. Each client *i* has a time window $[t_{ia}, t_{ib}]$ meaning the vehicle must arrive between $[t_{ia}, t_{ib}]$. In this problem the service time is very small, which means only the travelling time is considered. Before providing the mathematical formulation, we present here some notations and definitions used throughout the paper.

Mathematical notations:

C= $\{1,2,\dots,n\}$ - the set of n clients;

$N = \{0,1,2,\dots,n+1\}$ - the set of all nodes including the depot;

$V = \{1,2,\dots,v\}$ - the fleet of vehicles.

Decision Variables:

x_{ijk} - activate (1) or not (0) the connection between node *i* to *j* using vehicle *k*, $\forall\, i,j \in N, \forall\, k \in V$ with $i \neq j, i \neq n+1, j \neq 0$;

t_{ik} - time instant when vehicle *k* arrives to node *i*, $\forall\, i \in N, \forall\, k \in V$. It is assumed that t_{0k} is the referential for time =0 (departure of vehicle k from depot).

Parameters

$time_{ij}$ - time to travel from node i to j, for all the vehicles, $\forall\, i,j \in N$;

$[t_{ia}, t_{ib}]$ - time window for client i, $\forall\, i \in C$;

dem_i - product demand for client i, $\forall\, i \in C$;

q_k – capacity limit for vehicle k, $\forall\, k \in V$;

T – constant parameter to consider in the time window constraint $\rightarrow +\infty$;

2.1 Problem Formulation:

We are now in position to present our model mathematically. Our problem of interest is now formulated as the following

$$\text{Min } \alpha \sum_k^V \sum_i^N \sum_j^N x_{ijk} \cdot t_{ij} + \beta \sum_k^V \sum_j^N x_{0jk} \quad (1)$$

subject to:

$$\sum_k^V \sum_j^N x_{ijk} = 1 \quad \forall i \in C \quad (2)$$

$$\sum_j^N x_{0jk} = 1 \quad \forall k \in V \quad (3)$$

$$\sum_i^C dem_i \sum_j^N x_{ijk} \leq q_k \quad \forall k \in V \quad (4)$$

$$\sum_i^N x_{iak} - \sum_j^N x_{ajk} = 0 \quad \forall i \in C, \forall k \in V \quad (5)$$

$$\sum_i^N x_{i,n+1,k} = 1 \quad \forall k \in V \quad (6)$$

$$\sum_{i,j}^S x_{ijk} \leq |S| - 1 \quad S \subset C, 2 \leq |S| \leq n+2 \quad \forall k \in V \quad (7)$$

$$t_{ik} + time_{ij} - T(1 - x_{ijk}) \leq t_{jk} \quad \forall\, i,j \in N, \forall k \in V, T \rightarrow +\infty \quad (8)$$

$$ta_i \leq t_{ik} \leq tb_i \quad \forall i \in N, \forall k \in V \quad (9)$$

$$x_{ijk} \in \{0,1\} \quad \forall\, i,j \in N, \forall k \in V \quad (10)$$

$$t_{ik} \geq 0 \quad \forall i \in C, \forall k \in V \quad (11)$$

Now we will explain the justifications of taking the above model (1)-(11). In this formulation (1) represents the objective function, where α and β must reflect the weights for the two objective functions, that is why we proposed the model as multi objective. In this paper the main objective is to minimize the number of vehicles so we consider α=1 and β a big number to penalize the introduction of more vehicles. The total travelling time is to be minimized and the number of vehicles that departs from node 0 (depot) is to be penalized, in order to be minimum. The constraints (2) guarantee that only one vehicle visits each client. Constraints (3), (5) and (6) guarantee that each vehicle leaves the depot 0; after a vehicle arrives at a client, it must unload the goods to that client and then depart to the next one; and finally all vehicles arrive at depot (n+1). The constraint (4) indicates that the vehicle can be loaded only to its maximum capacity. The set of constraints in (7) have the goal of avoiding sub-cycles. The constraint (8) establishes the relationship between the departure time of the vehicle from a client and its immediate successor. The constraint (9) represents the time windows for each client and in (10) and (11) the decision variables.

3. Computational Tests and Results

Now we present some tests and results to a real problem of the *Just in Time Delivery* Company. The *Just in Time Delivery S.A.* needs to buy some vehicles to ensure the delivery of a service to a set of 6 important clients. The clients are geographically distributed in the north of Portugal like shown in Fig. 1:

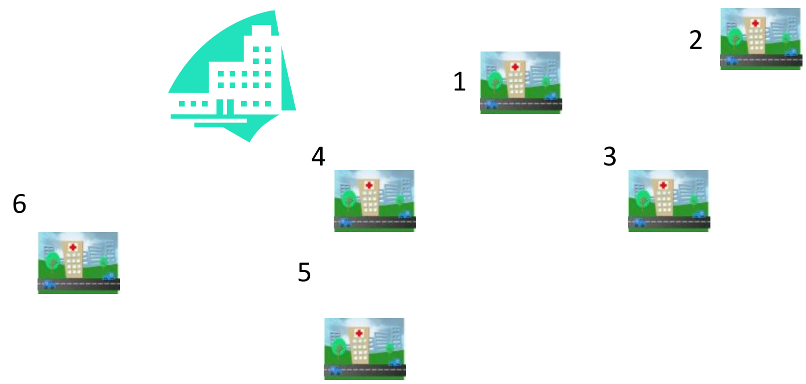

Fig. 1 Geographical distribution of the *Just in Time Delivery S.A.* clients.

The time distances, the time windows and the demand of the clients are presented in Table 1 and Table 2.

Table 1 : Time distance between the nodes.

Time units	Depot	1	2	3	4	5	6
Depot	-	30	60	50	25	50	40
1	30	-	30	25	25	50	80
2	60	30	-	35	25	60	100
3	50	25	35	-	50	30	75
4	25	25	25	50	-	25	45
5	50	50	60	30	25	-	55
6	40	80	100	75	45	55	-

Table 2 : Demand and time windows for each client.

Client	1	2	3	4	5	6
Demand (units)	10	30	5	25	10	10
t_a (time units)	10	60	50	25	50	40
t_b (time units)	100	70	150	150	60	200

The fleet of vehicles that the *Just in Time Delivery S.A* company wants to use has the capacity of 50 units each.

Next we present the results to this main problem, obtained by mathematical computation in GLPK. The testes were made in a 1.8 MHz PC with 512 MB of RAM.

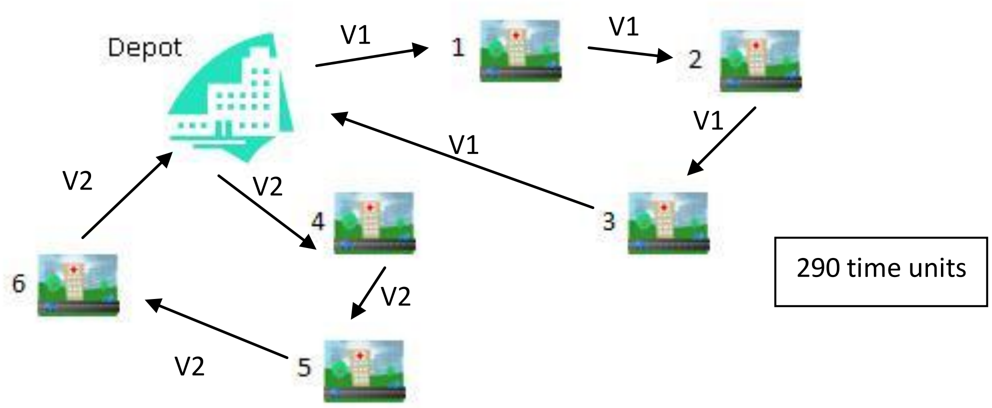

Fig. 2 Results for the main problem.

As shown in Fig. 2, the *Just in Time Delivery S.A.* only needs two vehicles to satisfy this client's demand. For that number of vehicles, the cost is of 290 time units.

The processing time for obtaining this solution was 210 seconds.

3.1 Sensibility Analyzes: vehicle capacity

One of the concerns of the *Just in Time Delivery S.A.* was the number of vehicles to use. So we made a sensibility analyzes by increasing the capacity of the vehicles to 100 units. The results are showed in Fig. 3 below.

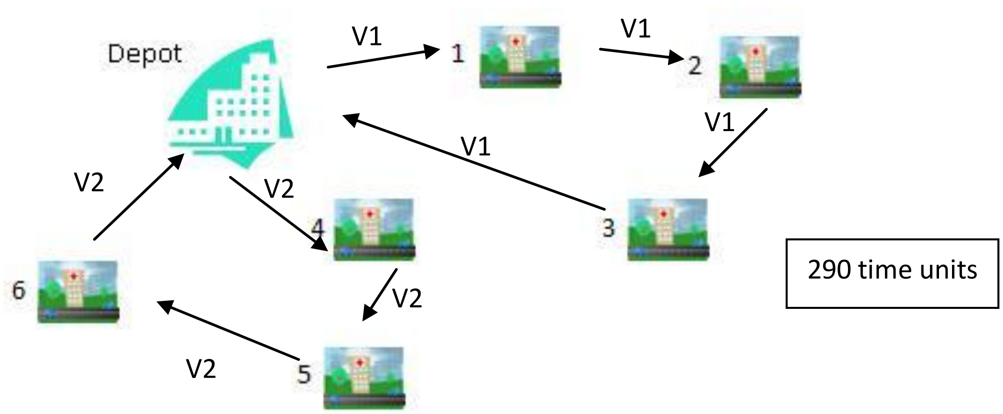

Fig. 3 Results for the sensibility to the vehicles capacity.

As shown in fig. 3 the results were the same. The company continues to need the same number of vehicles (two), despite one vehicle is sufficient to satisfy all the demands. This fact is related to the time window of client 2 and 5 that impose a very tight time interval for the vehicle to be there and unload the goods. Thus, we must conclude that the time windows are a very important constraint in these kinds of problems.

3.2 Sensibility Analyzes: time window of clients 6

The *Just in Time Delivery S.A.* has another concern with client 6, because it is possible that client 6 changes his time window. So it was made another sensibility analysis by changing the time window to t_{a6} = 60 and t_{b6} to 70 time units. The results are presented in Fig. 4 below.

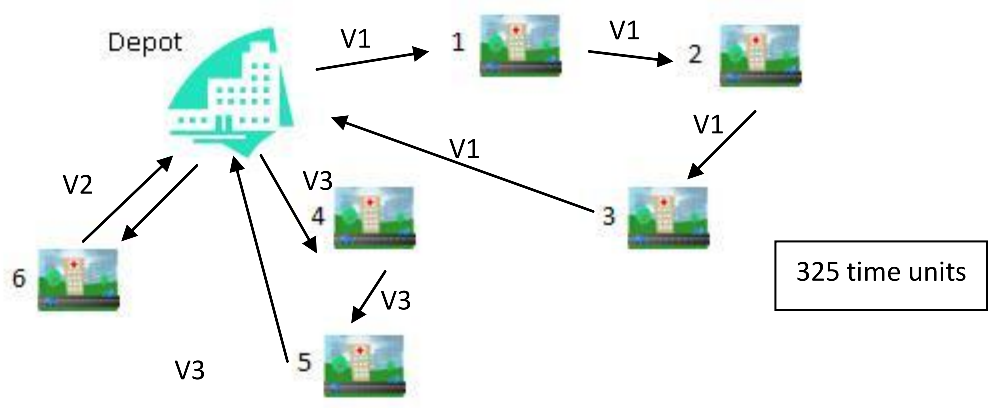

Fig. 4 Results for the sensibility to the vehicles capacity.

In this case it is necessary 3 vehicles because there are 3 clients with very tight time windows.

4. Conclusion

This paper has presented one multi objective approach to solve a Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) minimizing the number of vehicles in use and the total travelling time. The implemented process was tested in a real case scenario of *Just in Time Delivery S.A.* with good results in a reasonable time. It can be concluded that these kinds of problems are very hard in terms of computational effort. Thus, for further greater problems, to obtain a solution there must be applied other techniques like Heuristics or Meta Heuristics. It also can be concluded that time windows are very important in obtaining a solution for the problem. In order to obtain better results and eventually reducing even more the Total Travel Time it can be considered the possibility of a vehicle arrival to a customer before the Time Window. This can be done considering a Waiting Time t_w, so that t_w must be carefully chosen. Long waiting times should be avoided for reasons such as driver wages, driver satisfaction or the condition of the products. Taking into account this new variable, the first part of the objective function would be to minimize Total Used Time, where Used Time = Travel Time + Waiting Time.

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