An Immigrated Ecological Ammensalism with Limited Resources

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Abstract

The present paper brings out a mathematical model of Ammensal-enemy both immigrated species pair with limited resources. This model is built by a couple of first order non-linear ordinary differential equations. One and only one equilibrium point of this model is traced and the stability criteria at this equilibrium point are derived. Solutions for the linearized perturbed equations are ascertained.

Keywords: *Equilibrium point, Stability, Carrying capacity, Equilibrium state*

1. Introduction

Ecology being a science, synthetic in principle, concerns with a wide variety of models .A biological community, which exists for sufficiently long period in a more or less invariable state, should possess intrinsic abilities to resist perturbations coming in abundance from the environment. This ability of an ecosystem is usually termed as system-stability. A community is considered to be stable, if the number of member species remains constant over sufficiently long time intervals. On the other hand, an advanced theory of mathematical stability is available which deals with mathematical models of real objects. Therefore, if we have a good model of an ecosystem (in terms of differential or difference equations), the stability of real community can be deduced from our model by conventional methods of stability theory. For instance, the ecosystem may be considered to be stable, when the model trajectories of solutions of a system of equations in the phase space stay within a given bounded domain for a sufficiently wide range of perturbations.

Ecologists and mathematicians imparted to the growth of this area of knowledge reported as in the treatises of Lotka [12], Meyer [13], and Kapur [9, 10] N.C.Srinivas [14] studied the competitive ecosystems of multiple interacting species. Lakshminarayana and Pattabhi

Ramacharyulu [11] investigated prey-predator ecological models with a partial cover for the prey. The present authors [1-8] investigated several interactions of ecological Ammensalism between multiple interacting species with multifarious resources.

The present concept is aimed to study an analytical investigation of Ammensal-enemy both immigrated species pair with limited resources. The mathematical model is constructed by a couple of first order non-linear ordinary differential equations. The only existing equilibrium point is discovered and the stability criteria for it is discussed. Solutions for the linearised perturbed equations are established and results are laid out.

1.1. Notation Adopted

N_1 and N_2 are the populations of the Ammensal (S_1) and enemy (S_2) species with natural growth rates a_1 and a_2 respectively.
a_{11} is rate of decrease of the Ammensal due to insufficient food.
a_{12} is rate of increase of the Ammensal due to inhibition by the enemy.
a_{22} is rate of decrease of the enemy due to insufficient food.
$h_i = a_{ii} H_i$ is rate of immigration of the species S_i.
$K_i = a_i / a_{ii}$ are the carrying capacities of S_i, i = 1, 2
$\alpha = a_{12}/a_{11}$ is the coefficient of Ammensalism.
The state variables N_1 and N_2 as well as the model parameters a_1, a_2, a_{11}, a_{22}, K_1, K_2, α, h_1, h_2 are assumed to be non-negative constants.

2. Basic Equations

(i) Equation for growth rate of Ammensal species (S_1)

$$\frac{dN_1}{dt} = a_{11} [K_1 N_1 - N_1^2 - \alpha N_1 N_2 + H_1] \qquad (1)$$

(ii) Equation for growth rate of enemy species (S_2)

$$\frac{dN_2}{dt} = a_{22} [K_2 N_2 - N_2^2 + H_2] \qquad (2)$$

2.1. Equilibrium States

The system has only one co-existence state resulting from $\frac{dN_1}{dt} = 0;\ \frac{dN_2}{dt} = 0$

i.e. $\overline{N}_1 = \left(K_1 - \alpha\left(K_2 + \frac{H_2}{K_2}\right)\right) + \frac{H_1}{K_1 - \alpha\left(K_2 + \frac{H_2}{K_2}\right)}; \overline{N}_2 = K_2 + \frac{H_2}{K_2}$ (Co-existence state E_1)

2.1.1. Stability of the equilibrium states

Let $N = (N_1, N_2) = \overline{N} + U = (\overline{N}_1 + U_1, \overline{N}_2 + U_2)$ (4)

where $U = (U_1, U_2)$ is a perturbation over the equilibrium state $\overline{N} = (\overline{N}_1, \overline{N}_2)$. The basic equations (1) and (2) are linearized to obtain the equations

$$\frac{dU}{dt} = AU \qquad (5)$$

where $A = \begin{bmatrix} a_{11}(K_1 - 2\overline{N}_1 - \alpha\overline{N}_2) & a_{11}\alpha\overline{N}_1 \\ 0 & a_{22}(K_2 - 2\overline{N}_2) \end{bmatrix}$ (6)

The characteristic equation for the above system is

$$\det\,[A - \lambda I] = 0 \tag{7}$$

3. Stability of the Equilibrium State E_1:

$$\overline{N}_1 = \left(K_1 - \alpha\left(K_2 + \frac{H_2}{K_2}\right)\right) + \frac{H_1}{K_1 - \alpha\left(K_2 + \frac{H_2}{K_2}\right)};\overline{N}_2 = K_2 + \frac{H_2}{K_2}$$

From (5), the corresponding linearized perturbed equations are

$$\frac{d}{dt}\begin{bmatrix}U_1\\U_2\end{bmatrix} = \begin{bmatrix} -a_{11}\left[\frac{2H_1}{K_1-\alpha\left(K_2+\frac{H_2}{K_2}\right)} + \left(K_1-\alpha\left(K_2+\frac{H_2}{K_2}\right)\right)\right] & -\alpha a_{11}\left[\left(K_1-\alpha\left(K_2+\frac{H_2}{K_2}\right)\right) + \frac{H_1}{K_1-\alpha\left(K_2+\frac{H_2}{K_2}\right)}\right] \\ 0 & -a_{22}\left(K_2+\frac{2H_2}{K_2}\right) \end{bmatrix}\begin{bmatrix}U_1\\U_2\end{bmatrix} \tag{8}$$

The characteristic equation for the above system is

The $\lambda^2 + \lambda\left[a_{22}\left(K_2 + \frac{H_2}{K_2}\right) + a_{11}\left[\frac{2H_1}{K_1 - \alpha\left(K_2 - \frac{H_2}{K_2}\right)} + \left(K_1 - \alpha\left(K_2 + \frac{H_2}{K_2}\right)\right)\right]\right]$

$$+a_{11}a_{22}\left(K_2 + \frac{H_2}{K_2}\right)\left(\frac{2H_1}{K_1 - \alpha\left(K_2 + \frac{H_2}{K_2}\right)} + \left(K_1 - \alpha\left(K_2 + \frac{H_2}{K_2}\right)\right)\right) = 0 \tag{9}$$

The characteristic roots of (9) are

$$\lambda_1 = -a_{11}\left(\frac{2H_1}{K_1 - \alpha\left(K_2 + \frac{H_2}{K_2}\right)} + \left(K_1 - \alpha\left(K_2 + \frac{H_2}{K_2}\right)\right)\right);\ \lambda_2 = -a_{22}\left(K_2 + \frac{H_2}{K_2}\right)$$

both of which are negative. Hence the co existence State is **stable**.

By solving the system of equations in (8) we get

$$U_1 = \beta e^{-a_{22}\left(\overline{N_2} + \frac{H_2}{K_2}\right)t} + \left[U_{10} - \beta\right]e^{-a_{11}\left(\frac{H_1}{K_1 - \alpha\overline{N_2}}\right)t} + \tag{10}$$

where $\beta = \dfrac{\alpha a_{11} U_{20} \overline{N_1}}{a_{22}\left(\dfrac{H_2}{K_2} + \overline{N_2}\right) - a_{11}\left(\dfrac{H_1}{K_1 - \alpha \overline{N_2}} + \overline{N_1}\right)}$

$$U_2 = U_{20}\, e^{-a_{22}\left(\overline{N_2} + \frac{H_2}{K_2}\right)t} \qquad (11)$$

If $U_{10} = Q$, then the solutions become

$$U_1 = U_{10}\, e^{-a_{22}\left(\overline{N_2} + \frac{H_2}{K_2}\right)t} \text{ and } U_2 = U_{20}\, e^{-a_{22}\left(\overline{N_2} + \frac{H_2}{K_2}\right)t} \qquad (12)$$

The solution curves in (12) are illustrated as follows:

Case (i): $U_{10} < U_{20}$

The initial population strength of the Ammensal is less than that of the enemy i.e. $U_{10} < U_{20}$. In this case the enemy remains the dominance Ammensal as shown in Figure .1.

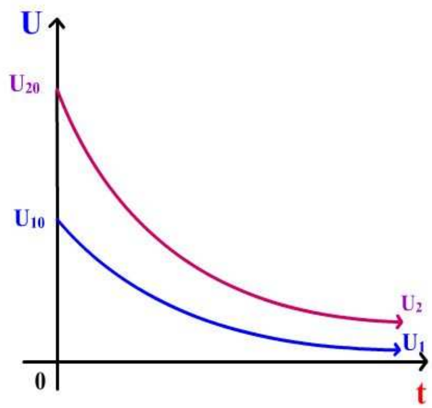

Figure .1.Asymptotic convergence of Ammnesal and enemy species

Case (ii): $U_{20} < U_{10}$

The initial population strength of the enemy is less than that of the Ammensal i.e. $U_{20} < U_{10}$. Initially the Ammensal prevails the enemy and this carries on up to the time $t = t^* = \dfrac{1}{a_{22}\left(\dfrac{H_2}{K_2} + \overline{N_2}\right) - a_{11}\left(\dfrac{H_1}{K_1 - \alpha \overline{N}_2} + \overline{N_1}\right)} \log\left(\dfrac{U_{10} - M}{U_{20} - M}\right)$ after which, the enemy eclipses Ammensal as illustrated in Figure .2.

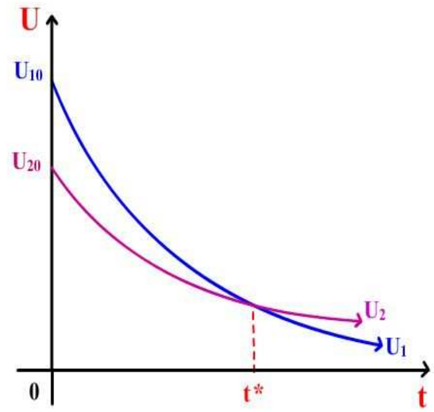

Figure .2. Asymptotic convergence of Ammnesal and enemy species with t*

3.1 Trajectories of perturbed species

By eliminating't' from the equations (10) and (11)

we get $\frac{U_1}{U_{10}} = -\frac{\beta}{U_{10}}\left(\frac{U_2}{U_{20}}\right) + \left(1+\frac{\beta}{U_{10}}\right)\left(\frac{U_2}{U_{20}}\right)^Q$ (13)

where $Q = \frac{a_{11}\left(\frac{H_1}{K_1 - \alpha \overline{N_2}} + \overline{N_1}\right)}{a_{22}\left(\overline{N_2} + \frac{H_2}{K_2}\right)}$ and these curves are shown in Figure .3 which exemplifies the stability of the equilibrium point.

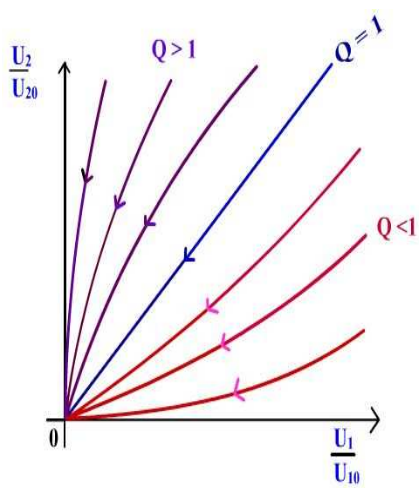

Figure .3.Stabilized species

4. Conclusion and Further work: The stability of a mathematical model of Ammensal-enemy both immigrated species pair with limited resources is explained and also it is observed that The model is stable at co-existence state.

For further work,One can investigate a model of four species (S_1, S_2, S_3, and S_4); S_1 and S_2 are neutral to each other, S_3 is a predator living up on S_1, S_4 is a predator living up on S_2 and S_3

&S_4 are co-operate with each other struggling for existence. In addtion to this, the harvesting concept can also be incorporated.

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