International Journal of Advanced Science and Technology
Vol. 16, March, 2010

An Improved Denoising Algorithm using Parametric Multiwavelets for Image Enhancement

Vidhyalavanya. R[1] and Madheswaran. M[2]

[1]*Amrita Vishwa Vidyapeetham,Coimbatore, India,*
[2]*Muthayammal Engineering College*
Rasipuram, India
r_vidhyalavanya@cb.amrita.edu, madheswaran.dr@gmail.com

Abstract

The problem of estimating a signal that is corrupted by additive noise has been of interest to many researchers for practical as well as theoretical reasons. Many of the traditional denoising methods have been using methods such as the Wiener filtering. Recently, nonlinear methods, especially those based on wavelets have become increasingly popular, due to a number of advantages over the linear methods. It has been shown that wavelet and multiwavelet thresholding guarantees better rate of convergence, despite its simplicity. This paper demonstrates the work of combining Parametric multiwavelet and Sureshrink to remove noise from the signal. Experimental results shows that the proposed work is 4% efficient in terms of SNR values and image quality when compared to other wavelet families

Keywords: *Parametric Multiwavelet, Denoising, Sureshrink, Pre and post filtering.*

1. Introduction

Signal available in the real world are always corrupted with noise. Noise is an unavoidable signal during transmission. Many researches concentrate on the removal of noise from the signal. The process of removing noise from the signal is called as denoising. Under ideal condition, this noise may decrease to such negligible levels while the signal will increase to a significant level. Removal of noise actually started in the time domain. The extraction of pure signal from corrupted signal is not appreciable in time domain, whereas if the same done in frequency domain the performance was better. Therefore the researchers use the frequency domain rather than time domain. For conversion if the Fourier Transform is used the perfect removal is not possible. For the last 15 years wavelet shrinkage method is used which do the job more efficiently than most other methods in denoising. Three major steps are followed for the process of denoising. They are

1. A linear forward wavelet transform
2. A non-linear shrinkage denoising
3. A linear inverse wavelet transform

In the recent years there has been a fair amount of research on wavelet thresholding and threshold selection for signal de-noising [1-4] because wavelet provides an appropriate basis for separating noisy signal from the image signal. The motivation is that as the wavelet transform is good at energy compaction, the small coefficients are more likely due to noise and large coefficient due to important signal features [5].

Donoho and Johnstone proved several important theoretical results on wavelet thresholding, or wavelet shrinkage [6-7]. They showed that wavelet shrinkage has many excellent properties, such as optimality in minima sense, and a better rate of convergence [6-7]. DeVore and Lucier have also arrived at the wavelet thresholding concept, starting from their independent work on variation problems [8].

Wavelet shrinkage depends heavily on the choice of a thresholding parameter and the choice of this threshold determines, to a great extent the efficiency of denoising. The denoising process is based on the fact that the wavelet transform compresses most of the L^2 energy of the signal in a restricted number of large coefficients. The procedure can be summarized in three steps

$$Y=W(X)$$
$$Z=T(Y,\lambda) \quad (1)$$
$$Y^1=W^{-1}(Z)$$

where x is the affected signal, W (.) and W^{-1} is the forward and inverse wavelet transform operators. T(Y, λ) denotes the denoising operator with soft or hard threshold [l]. Of the various methods based on wavelet thresholding, TopShrink [9], SureShrink [10], BayesShrink [11] and its variants are the most popular. VisuShrink uses one of the well known thresholding rules: the universal threshold. In addition, subband adaptive systems have superior performance, such as SureShrink, which is a data driven system. Recently, SureShrink [11], which is also a data driven subband adaptive technique, is proposed and outperforms TopShrink and BayesShrink. In the proposed method SureShrink is used along with anisotropic diffusion to get a better performance than stand alone anisotropic diffusion or BayesShrink.

It is already clear that suppression of speckle noise is necessary to get reliable measurements. At the moment we are researching new filtering techniques in order to remove this speckle noise as much as possible and preserve details as well. In this article we compare three noise removal techniques, based on wavelet decomposition, applied to speckle images. In this paper Parametric multiwavelet is used, which exhibit good frequency resolution and compact support

2. Background

2.1 Denoising using wavelet shrinkage-Statistical modelling and estimation.

Consider the standard univariate nonparametric regression setting

$$X_i(t) = S_i(t) + \sigma\varepsilon_i(t), \quad i = 1,2,\ldots,n \quad (2)$$

Where $X_i(t)s$ are assumed to come from zero-mean Normal distribution, ε_i are independent standard normal - N (0, 1) - random variables and noise level σ may be known or unknown. The goal is to recover the underlying function S from the noisy data, $X = (X_1, X_2, \ldots, X_n)'$ without assuming any particular parametric structure for.. S

For images, the model is

$$X_{i,j}(t) = S_{i,j}(t) + \sigma\varepsilon_{i,j}(t), \quad i = 1,2,\ldots,I, \quad j = 1,2,\ldots,J, \; \varepsilon_{i,j} \sim N(0,1) \tag{3}$$

The three main steps of denoising using wavelet coefficient shrinkage technique are as follows

1) Calculate the wavelet coefficient matrix w by applying a wavelet transform W to the data:

$$w = W(X) = W(S) + W(\sigma\varepsilon), \tag{4}$$

2) modify the detail coefficients (wavelet coefficients) of w to obtain the estimate $\hat{w}$ of the wavelet coefficients of *S:*

$$w \to \hat{w} \tag{5}$$

3) Inverse wavelet transform for the modified detail coefficients to obtain the denoised coefficients

$$\hat{S} = W^{-1}(\hat{w}) \tag{6}$$

The number n of the wavelet coefficients w in Equ. (4) varies depending on the type of transform (decimated or undecimated) used. w consists of both scaling coefficients and wavelet coefficients. In decimated wavelet transform, the number of coefficients in w is same as number of data points. There will be n/2 scaling coefficients and equal number of wavelet coefficients in w.

The first step in denoising is to select a wavelet for the forward and inverse transformation W and W^{-1}.

There are variety of wavelets that can be used which differs in their support, symmetry, and number of vanishing moments. In addition to a wavelet, we also need to select number of multiresolution levels and the option for handling values near the edge of the image. There are several boundary treatment rules including periodic, symmetric, reflective, constant and zero-padding. If the selections of filters are perfect then denoising can also be applied without problems in cardiac imaging also. In the recent years there has been increasing number of research activities on wavelets and multiwavelets for applications like denoising. This lead to the development of many algorithms for removal of noise from the signal.

2.2 Parametric Multiwavelets

Capacity to represent localized phenomena, represent variables and seek solutions to a predetermined level of resolution and use of computing power are the features in their favour [12-13]. The scalar wavelets have only one scaling function and N-1 wavelet functions. They failed to satisfy the orthogonal, symmetric, antisymmetric and biorthogonal properties simultaneously [14]. Since multiwavelets has more than one scaling functions, it is possible to use correct stencils and was able to identify the low and high frequency in a better way [15]. Multiwavelets with vanishing moments maintain convergence of order M-1 upto the boundary-a unique property not shared by the scalar wavelet [15]. The interpolating property of the multiwavelet basis makes the coefficient values same as the values of the solution, thereby reducing the computational overhead. The use of a set of short support filters in multiwavelet leads to dual benefits over scalar wavelets. The first one is that multiwavelet with a given support can achieve the smoothness offered by scalar wavelets with larger support. The second benefit is that multiwavelet provides better compaction than the scalar wavelets [15]. Availability of a large number of wavelet families implies a corresponding high level of flexibility in the use of image compression. However the number of multiwavelet families available is limited putting a corresponding restriction on the possibilities of compression application. Parametric multiwavelets have the advantage that the user can optimize the multiwavelet system for any application. It is possible to generate scaling function coefficients by varying angular parameters. A method for the construction of the parametric multiwavelet [15] has been formulated. It was reported in [15] parametric multiwavelet based transforms exhibit good frequency resolution, compact support, orthogonality, arbitrary approximation order and symmetry at the same time.

For α = 0, the symbol of the symmetrical cardinal B-spline of order 2 is obtained. The cardinal B-splines are the most regular refinable functions with respect to their supports. The support of Φ_α^2 is contained in [-1, 1]. Multiwavelets as an extension of scalar wavelets have received considerable attention recently from wavelet research communities. Multiwavelets can be considered as a system of wavelets with more than one scaling and wavelet functions.

3. Proposed work

3.1 Parametric multiwavelet with Sure shrink

For the coefficients of parametric multiwavelets in equation (9) the wavelet transform is applied using preprocessing and post processing filter. The filter used for it is Hardin-Roach filter.

The matrix coefficient $\{H_k\}$, $\{G_k\}$ are of the form

$$[H_k] = \begin{bmatrix} h_0(2k) & h_0(2k+1) & \ldots\ldots\ldots & h_0(2k+r-1) \\ h_1(2k) & h_1(2k+1) & \ldots\ldots\ldots & h_1(2k+r-1) \\ . & & & \\ . & & & \\ h_{r-1}(2k) & h_{r-1}(2k+1) & \ldots\ldots\ldots & h_{r-1}(2k+r-1) \end{bmatrix} \quad (7)$$

$$[G_k] = \begin{bmatrix} g_0(2k) & g_0(2k+1) & \ldots\ldots\ldots & g_0(2k+r-1) \\ g_1(2k) & g_1(2k+1) & \ldots\ldots\ldots & g_1(2k+r-1) \\ . & & & \\ . & & & \\ g_{r-1}(2k) & g_{r-1}(2k+1) & \ldots\ldots\ldots & g_{r-1}(2k+r-1) \end{bmatrix} \quad (8)$$

The [illegible] multiwavelets of multiplicity r = 2 and approximation order $\kappa \geq 1$ is given here where κ is the approximation order and α is the parameter.

Computation of ${}_{\kappa}A_{\alpha}(z)$ involves three steps [15]:

1. Defining general symbol entries.
2. Eigen value condition.
3. Factorization condition.

The symbol A as the scaling coefficients, for k = 2can be written as

$$ {}_2A_\alpha(z) = \begin{pmatrix} 1 & \left(\frac{1}{2}-\alpha\right)z^{-1}+\frac{1}{2}+\alpha z \\ z & \alpha z^{-1}+\frac{1}{2}+\left(\frac{1}{2}-\alpha\right)z \end{pmatrix} \quad (9) $$

Once the filter coefficients are found out for a particular value of α , the w_k – detail coefficients are obtained. Using these values denoising process is started. After performing denoising using the same set of filter coefficients the inverse multiwavelt transform is applied to get back the pure signal.

Donoho and Johnstone proposed a robust estimate of the noise level σ given by

$$ \sigma = \text{median } \{(w_k: k=1, 2,....,n/2)\}/0.6745 \quad (10) $$

Here w_ks are detail coefficients at the finest level. Let w denote a single detail coefficient and w^1 denote its shrink version. Let λ be the threshold and D^λ (.) denote shrinkage function which determines how threshold is applied to the data and σ^1 be the estimate of the standard deviation of the noise, then

$$ W^1=\sigma^1.D^\lambda(w/\sigma^1) \quad (11) $$

By dividing w with σ^1 the w coefficients are standardized to get w_s and the threshold operator is applied. After thresholding, the resultant coefficients are multiplied with σ^1 to obtain w^1. If σ^1 is build into the threshold model or if the data is normalized with respect to noise standard deviation, equation for estimated value of w is:

$$ W^1=D^\lambda(w) \quad (12) $$

The first part of the proposed work is to find out the threshold value using SURE (Stein Unbiased Risk Estimate). The main advantage of selecting SURE shrinkage rule is that the generalization of images can be achieved in either level- or subband-dependent manner. In the latter case, the threshold on subband S is

$$ \lambda_s = \arg(\min \lambda>=0[SURE^s(\lambda,w_s) \quad (13) $$

Where w_s denotes the detail coefficients from the subband S and SURE (λ,w_s) denote the corresponding Stein's unbiased estimate of the risk corresponding to a specific shrinkage function.

$$ SURE^S[\lambda,w_s]=N_s + \sum [\min (w_k),\lambda]^2 - 2[w_k] \quad (14) $$

It was shown by Donoho and Johnstone that, in case where the wavelet coefficients decomposition is sparse, a hybrid method combining the universal and SURE thresholds is preferable over SURE. This hybrid method, when combined with soft shrinkage function is referred to as Sure Shrink.

The shrinkage function determines how the thresholds are applied to the data. The mathematical expression for soft threshold is

$D^{\lambda}(w) = \mathrm{sgn}(w)\, \max(0; w-\lambda)$ (15)

//Algorithm to perform denoising using Sureshrink

//Inputs

W_k: detail coefficients

N_s: Number of coefficients

//Outputs

λ:threshold

//Computations

Load a 2-D noisy image

{

Fix the noise standard deviation σ

Perform Wavelet transform using the coefficients from equation (9)

Calculate the value of λ from the equations (13) and (14)

For the value of λ find $D^{\lambda}(.)$ using the equation (15)

Perform inverse wavelet transform for the same set of coefficients in equation (9)

Find out the difference between the original and reconstructed image.

Find Signal to noise ratio

}

Repeat for different images and different values of σ Algorithm can be repeated for different coefficient set of parametric multiwavelet by changing the value of α in equation (9).

4. Results and discussion

The experiments are conducted on several natural gray scale test images like Lena and Barbara 256 × 256 at different noise levels σ=10, 20. The wavelet transform employs parametric multiwavelets compactly supported wavelet with eight vanishing moments [14] at four scales of decomposition. To assess the performance of TopShrink, it is compared with SureShrink, BayesShrink. To benchmark against the best possible performance of a threshold estimate, the comparison include BayesShrink, the best soft thresholding estimate obtainable assuming the original image known. The SNR values from various methods are compared in Table I and the data are collected from an average of five runs. Since the main comparison is against SureShrink and BayesShrink, the better one among these is highlighted in bold font for each test set. SureShrink outperforms Topshrink and BayesShrink most of the time in terms of SNR as well as in terms of visual quality. Moreover SureShrink is 4% better than

BayesShrink. The choice of soft thresholding over hard thresholding is justified from the results of best possible performance of a hard threshold estimator.

Comparisons are also made with the best possible linear filtering technique i.e. Hardin-roach filter. The results in the table I show that SNR are considerably worse than the nonlinear thresholding methods, especially when σ is large.

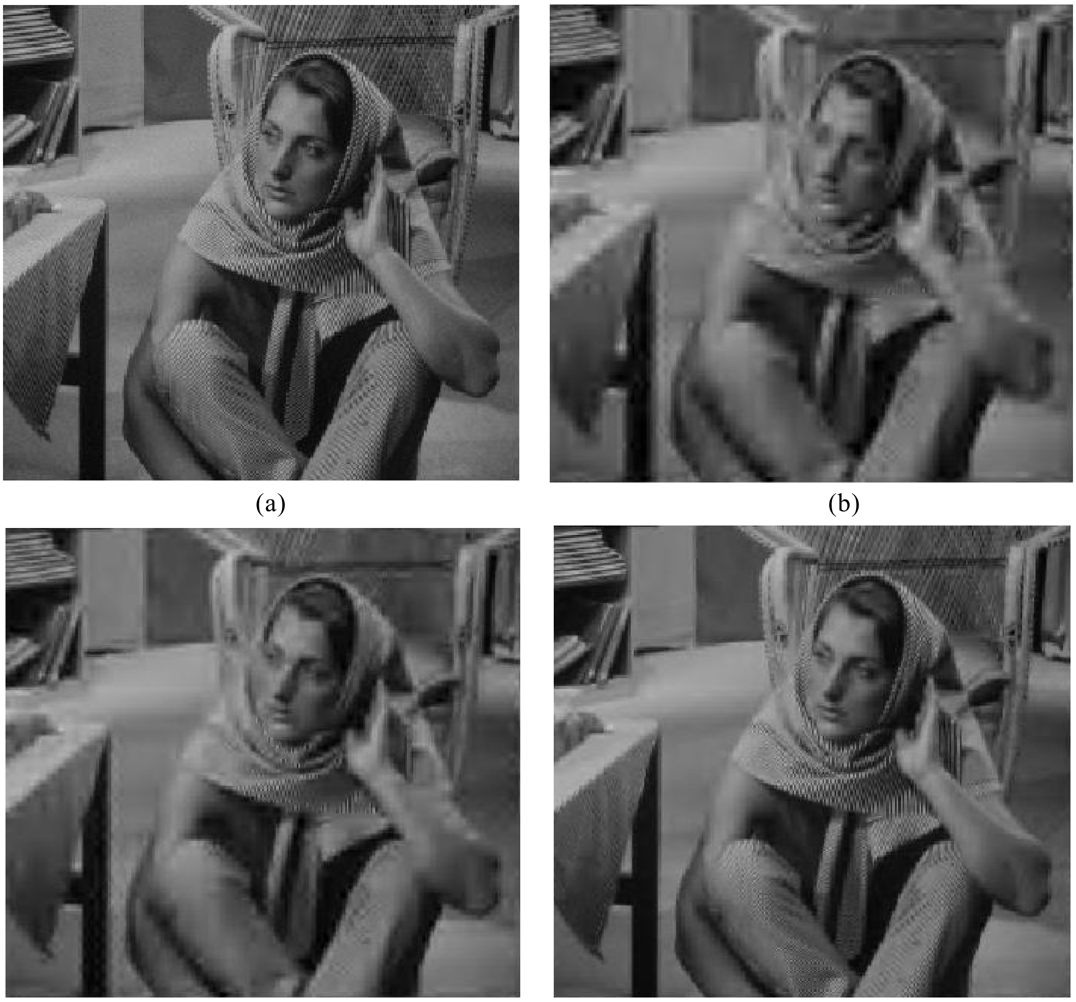
The image quality is also not as good as those of the thresholding methods.

Figure. 1. shows the image denoised with proposed multiwavelet, Sureshrink. It is observed that the proposed method improves the image quality. It can be seen that there is 10% improvement over other multiwavelets and around 5% over Bayes shrink in preserving image structure. Based on SNR also it can be seen that the proposed method performs better than the other two. Fig. 4 shows comparative analysis of GHM multiwavelet, bayesshrink and proposed method.

Table I. SNR results for various test images and σ values

	Topshrink (db)	Bayesshrink (db)	**Sureshrink (db)**
Barbera			
σ=10	45.34	45.87	**46.07**
σ=20	40.57	40.92	**41.21**
Lena			
σ=10	46.34	46.99	**47.23**
σ=20	43.68	43.98	**44.41**

It is clear that the performance of the methods depends on image type and noise levels. But in both cases, whether GHM multiwavelet or bayesshrink gives worst results, the performance of the proposed method seems to be much better than the other two. It can be seen that the proposed method preserves image structures much better than GHM multiwavelet and bayesshrink. Also the number of iterations required for the proposed method to produce the better image is much less than that of GHM multiwavelet. The experiment is repeated for various types of images with varying noise levels and seems that the method proposed is giving better results than GHM multiwavelet and Bayes shrink. It is clear that the performance of the methods depends on image type and noise levels. But in both cases, whether GHM multiwavelet or bayesshrink gives worst results, the performance of the proposed method seems to be much better than the other two. It can be seen that the proposed method preserves image structures much better than GHM multiwavelet and bayesshrink. Also the number of iterations required for the proposed method to produce the better image is much less than that of GHM multiwavelet. The experiment is repeated for various types of images with varying noise levels and seems that the method proposed is giving better results than GHM multiwavelet and Bayes shrink.



(a) (b) (c) (d)

Figure. 1. (a) Original Barbera image (b) Noisy Barbera of σ=10.
Reconstructed images (c) DGHM (45.83dB), (d) Proposed filter (46.07 dB).

5. Conclusion

In this paper, a parametric multiwavlet with sureshrink threshold is proposed to address the issue of image recovery from its noisy counterpart. It is based on the generalized Guassian distribution modeling of subband coefficients. The image denoising algorithm uses soft thresholding to provide smoothness and better edge preservation at the same time. Experiments are conducted to assess the performance of SureShrink in comparison with the Topshrink and Bayeshrink. The results show that SureShrink removes noise significantly and remains within 4% of Topshrink and outperforms BayesShrink. It is further suggested that the proposed threshold may be extended to the compression framework, which may further improve the denoising performance. Due to the inherent characteristic of the wavelet transform, we used parametric multiwavelets for restoration, making the proposed scheme computationally efficient. The comparative study of peak signal-to noise ratio with varying levels of noise intensities shows the improved reconstruction quality. Another advantage of the proposed scheme is that denoising in the image is found to improve the SNR of the whole image.

References

[1] A.B. S D.L. Donoho, De-Noising by Soft Thresholding, *IEEE Trans. Info. Theory 43*, pp. 933-936, 1993.

[2] S. Grace Chang, Bin Yu and M. Vattereli, Adaptive Wavelet Thresholding for Image Denoising and Compression, *IEEE Trans. Image Processing*, vol. 9, pp. 1532-1546, Sept. 2000.

[3] D.L. Donoho and I.M. Johnstone, Wavelet shrinkage: Asymptopia?, *J.R. Stat. Soc. B, ser. B*, Vol. 57, no. 2, pp. 301-369, 1995.

[4] M. Lang, H. Guo and J.E. Odegard, Noise reduction Using Undecimated Discrete wavelet transform, IEEE Signal Processing Letters, 1995.

[5] Maarten Jansen, Noise Reduction by Wavelet Thresholding, Springer –Verlag New York Inc.- 2001.

[6] Donoho, D. L. and Johnstone, I. M., "Ideal spatial adaptation via wavelet shrinkage", Biometrika, vol. 81, pp. 425- 455, 1994.

[7] Donoho, D. L., "De-noising by soft-thresholding", IEEE Trans. Info. Theory, vol. 41, pp. 613-627, May 1995.

[8] DeVore, R. A. and Lucier, B. J., "Fast wavelet techniques for near-optimal image processing", IEEE Military Communications Conference Record, San Diego, Oct. 11-14, 1992, vol. 3, pp. 1129-1135.

[9] Crouse, M. S., Nowak, R. D., Baraniuk, R. G., "Wavelet based signal processing using hidden Markov models", IEEE Trans. Signal Proc., vol. 46, pp. 886-902, Apr. 1998.

[10] David L. Donoho and Iain M. Johnstone, "Adapting to Unknown Smoothness via Wavelet Shrinkage," Journal of American Statistical Association, 90(432):1200-1224, December 1995

[11] Gao, H.-Y., "Wavelet shrinkage denoising using the nonnegative garrote", J. Comput. Graph. Statist., 7 (4), pp. 469- 488, 1998.

[12] D. P. Hardin and D. W. Roach, "Multiwavelet prefilter I: Orthogonal prefilters preserving approximation order p _ 2," IEEE Trans. Circuits Syst. II, vol. 45, pp. 1106–1112, Aug. 1998.

[13] C. Heil, G. Strang, and V. Strela, "Approximation by translates of refinable functions," Numer. Math., 1996.

[14] Xiang-Gen Xia, Jeffrey S. Geronimo, Douglas P. Hardin, and Bruce W. Suter. (1995). On computations of multiwavelet transforms. Proc. SPIE, 2569:27–38.

[15] Shen, L. X., Tan, H. H. and Tham, J. Y., "Symmetric-antisymmetric orthogonal multiwavelets and related scalar wavelets", Appl. Comput. Harmon. Anal., Vol. 8, 2000, pp. 258-279.

Authors

Vidhyalavanya R received her BE Degree from Madras University in 2001, ME Degree from National Institute of Technology, Calicut, Kerala, India in 2006 in Electronics and Communication Engineering. She is currently pursuing her PhD degree in Electronics Engineering from the Anna university, Coimbatore, Tamilnadu, India. She is working as Assistant professor in ECE department, Amrita Vishwa vidyapeetham, Ettimadai, Coimabtore. She has authored many research publications in International and National Journals and Conferences. Her areas of interest are VLSI signal processing, and Embedded systems. She is a member of Institution of Electronics and Telecommunication Engineers.

M.Madheswaran received the BE Degree from Madurai Kamaraj University in 1990, ME Degree from Birla Institute of Technology, Mesra, Ranchi, India in 1992, both in Electronics and Communication Engineering. He obtained his PhD degree in Electronics Engineering from the Institute of Technology, Banaras Hindu University, Varanasi, India, in 1999. At present he is a Principal of Muthayammal Engineering College, Rasipuram, India. He has authored over Seventy five research publications in International and National Journals and Conferences. Currently he is the chairman of IEEE India Electron Devices Society Chapter. His areas of interest are theoretical modeling and simulation of high-speed semiconductor devices for integrated optoelectronics application, Bio-optics and Bio-signal Processing. He was awarded the Young Scientist Fellowship (YSF) by the State Council for Science and Technology, TamilNadu, in 1994 and Senior Research Fellowship (SRF) by the Council of Scientific and Industrial Research (CSIR), Government of India in 1996. Also he has received YSF from SERC, Department of Science and Technology, Govt. of India. He is named in Marquis Who's Who in Science and engineering in the year 2006. He is a Member of Institute of Electrical and Electronics Engineers, Fellow of Institution of Electronics and Telecommunication Engineers, Member of Indian Society for Technical Education and Member of Institution of Engineers.