International Journal of Advanced Science and Technology
Vol. 14, January, 2010

Biomedical Signal Compression based on Basis Pursuit

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Abstract

This paper presents a discussion concerning EEG signals compression using the basis pursuit (BP) approach applied for several overcomplete wavelet dictionaries. The compression is based on an "optimal" superposition of dictionary elements, by minimizing the l1 norm of the error. The best results have been obtained with the Daubechies10 dictionary.

Keywords: *Electroencephalography, Basis Pursuit, Compression*

1. Introduction

Compression methods have gained in importance in recent years in many medical areas like telemedicine, health monitoring, etc. All these imply storage, processing, and transmission of large quantities of data. Compression algorithms can be based on direct methods, linear transformations, and parametric methods and can be classified into two main categories: lossless and lossy.

Even though many compression algorithms have been reported so far in the literature, not so many are currently used in monitoring systems and telemedicine.

The electrocardiogram (ECG) was introduced into clinical practice more than 100 years ago by Einthoven. It provides representation of the electrical activity of the heart over time and is probably the single-most useful indicator of cardiac function. It is widely accepted that the ECG waveforms reflect most heart parameters closely related to the mechanical pumping of the heart and can be used to infer cardiac health. The ECG waveform is recorded from the body surface using surface electrodes and an ECG monitoring system.

Electroencephalography (EEG) is the recording of electrical activity along the scalp produced by the firing of neurons within the brain and has been used for almost 70 years as a relatively inexpensive, noninvasive method for studying human brain function. Today, EEG has become one of the useful signals for clinical analysis, i.e. to diagnose the disease and to assess the effectiveness of the treatment via the brain functions. In clinical contexts, EEG refers to the recording of the brain's spontaneous electrical activity over a short period of time, usually 20–40 minutes, as recorded from multiple electrodes placed on the scalp. In neurology, the main diagnostic application of EEG is in the case of epilepsy, as epileptic activity can create clear abnormalities on a standard EEG study.

A long recording session of EEG acquisition is often required, especially in experiments which involve repeated stimuli and signal averaging, in sleep studies and in the monitoring of epileptic patients (ambulatory EEG). This, together with the multichannel nature of EEG causes the data files to be extremely large.

For electrodes placed on the scalp of a subject, electrical activity of the brain manifests itself as potential changes in the range of 0–200 μV, with a frequency range from less than 0.1 to 60 Hz or more. An EEG acquisition system (10–20 system) for clinical applications uses a number of 21 electrodes. Usually, EEG data is still sampled at a 100-Hz rate and 8-bit accuracy, although 200 Hz and 12 bits are recommended.

Taking into account that every sample of the EEG signals is very important and cannot be neglected without its consideration by experts, legal storage of such long-term EEG signals for further analysis has to be made either losslessly or using appropriate lossy compression methods.

As for any other signal family, EEG and ECG compression algorithms are supposed to achieve a reduced information rate, while retaining all relevant information in the reconstructed signal. A major aspect regarding signal compression is that of finding the most appropriate decomposition of the signal into orthogonal or linear independent signals.

In this paper the basis pursuit (BP) method for achieving EEG and ECG signal compression is investigated aiming at the best wavelet representation.

2. Background

Last years are characterized by an increased interest in alternatives to traditional signal representations. Besides representing signals as superpositions of sinusoids (the traditional Fourier representation), alternate dictionaries like wavelets, steerable wavelets, segmented wavelets, Gabor dictionaries, multi-scale Gabor dictionaries, wavelet packets, cosine packets, chirplets, warplets, and a wide range of other dictionaries are now available. For the discrete case, each such a dictionary $\Phi \in R^{MxN}$ is a collection of waveforms represented by discrete-time signals, called atoms. For a discrete-time signal $s \in R^{M}$, we envisage the decomposition of s as linear combination of dictionary atoms with corresponding coefficients $\alpha \in R^{N}$ in the form:

$$s = \Phi \alpha$$

or an approximate decomposition:

$$s = \Phi \tilde{\alpha} + \varepsilon$$

where ε is the approximation error.

Dictionaries can be complete or overcomplete, depending on the fact that they contain exactly N atoms, or more than N waveforms respectively. Undercomplete dictionaries containing less than N atoms are also used for special purposes.

Most of the new dictionaries are overcomplete, either because they start out that way, or because they are obtained by merging complete dictionaries, leading to new mega-dictionaries consisting of several types of waveforms (e.g. Fourier & wavelets dictionaries).

For example, the Haar dictionary is a collection of translations and dilatation of basic father and mother wavelets. The standard Haar dictionary consists of N waveforms. An overcomplete wavelet dictionary is obtained by using extra Haar-type signals potentially expressible with simpler Haar functions. A Haar wavelet overcomplete dictionary consists of $N' = N\log_2(N)$ waveforms (which is the dimension of α as well).

The decomposition of a signal using overcomplete dictionaries is thus nonunique, since some elements in the dictionary have representations in terms of other elements. It is thus possible to choose the most economic representation in terms of atoms.

To obtain signals representations in overcomplete dictionaries, several methods, as the method of frames [1], matching pursuit [2], basis pursuit (BP) [3] and method of best orthogonal basis [4] have been proposed in last years.

The method of basis pursuit (BP) finds the best representation of a signal by minimizing the l1-norm of α. Ideally, we would like as many components of α to be zero or as close to zero as possible. Formally, one solves the problem:

$$\text{minimize } \|\alpha\|_1 \quad \text{subject to } \Phi\alpha = s$$

The nonzero components of α correspond to the dictionary waveforms that will be used in the signal representation. Using the 11-norm allows to assign a cost to each atom used in the representation. For example, the norm will not be changed for zero coefficients, and will be changed proportionally for nonzero ones.

Since there is an additional condition to solving the system of equations, the signal decomposition can be viewed as a linear programming problem (LP) of the form:

$$\text{minimize } c^T\alpha \quad \text{subject to} \quad \Phi\alpha = s, \quad \alpha \in \mathrm{R}^{N'}$$

where $c^T\alpha$ is the objective function, $s = \Phi\alpha$ can be viewed as a collection of constraints.

To solve the previous equations any LP algorithm can be used - in this paper the Interior Point Method (IPM) has been chosen [6].

3. Methodology

Starting from the representation (1) a lossy compression of a signal can be achieved retaining only those coefficients with large values. In fact, the signal s is approximated by the relationship:

$$\Phi\tilde{\alpha} = \tilde{s}$$

where $\tilde{\alpha}$ is the vector with lowest l1 norm obtained using the LP algorithm.

The compression ratio (CR) is defined as the ratio between the number of bits needed to represent the original and the compressed signal.

For lossy compression techniques, the definition of the error criterion to appreciate the distortion of the reconstructed signal with respect to the original one is of paramount importance, particularly for biomedical signals like EEG and ECG signals, where a slight loss or modification of information can lead to wrong diagnostics. The measurement of these distortions is a difficult problem and it is only partially solved for biomedical signals. In most EEG and ECG compression algorithms, the percentage root-mean-square difference (PRD) measure defined as:

$$PRD\% = 100\sqrt{\frac{\sum_{n=1}^{M}(x(n)-\tilde{x}(n))^2}{\sum_{n=1}^{M}x^2(n)}}$$

is employed, where $x(n)$ is the original signal, $\tilde{x}(n)$ is the reconstructed signal, and M is the length of the signal. The normalized version of PRD, PRDN, which does not depend on the signal mean value, $\bar{x}$, is defined as:

$$PRDN\% = 100\sqrt{\frac{\sum_{n=1}^{M}(x(n)-\tilde{x}(n))^2}{\sum_{n=1}^{M}(x(n)-\bar{x})^2}}$$

Other measures such as the root mean square error (RMS):

$$RMS = \sqrt{\frac{\sum_{n=1}^{M}(x(n)-\tilde{x}(n))^2}{M}}$$

and the signal to noise ratio (SNR) are used as well [5]:

$$SNR = -20\log(0.01 * PRDN)$$

In all cases, the final verdict regarding the fidelity and clinical acceptability of the reconstructed signal should be validated through visual inspection by the cardiologist physician.

4. Experimental results and Discutions for EEG signals

We have used the WaveLab and the Atomizer software from MATLAB [6,7]. WaveLab is a library of MATLAB routines for wavelet, wavelet packet, and cosine packet analysis. Atomizer contains a collection of dictionaries and artificial signals. It borrows routines from WaveLab and includes codes for several methods for finding signal representations in overcomplete dictionaries [7] [8].

In order to compare the performances of various overcomplete dictionaries the record number s030 (the recording during the crisis from areas with ictal activity) consisting of the first 256, respectively the record number z003 (healthy patient –EEG recording with open eyes) consisting of the first 1024 samples from the Clinic of Epileptology of the

University Hospital of Bonn databases have been used. The EEG signals were digitized through sampling at 173,61 samples/s, quantized and encoded with 12 bits [8,9].

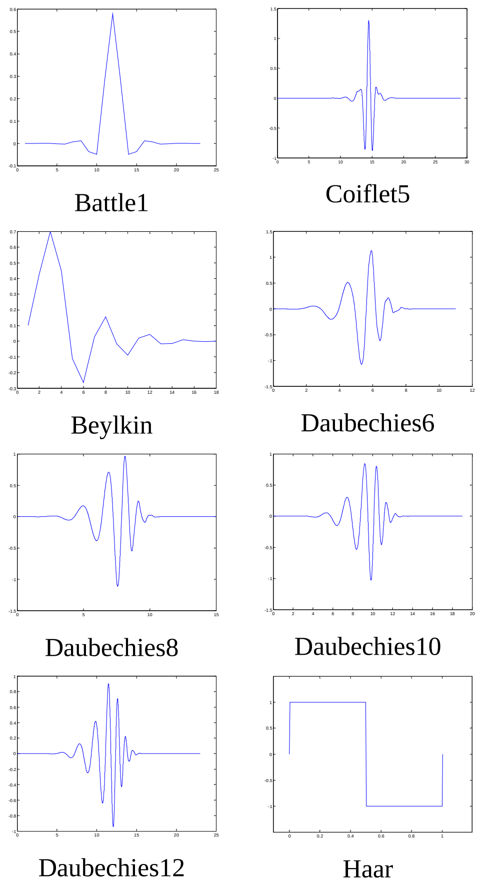

Figure 1. Mother wavelets used for compression

As can be seen from Table 1, the best results from the reconstruction error point of view were obtained using the Daubechies10 wavelet and the worst, using the Haar wavelets.

Table 1. Results of the compression algorithm for various dictionaries when 50 coefficients were retained

	CR	PRDN	PRD	RMS	SNR
Battle 1	5	18.94	18.84	77.38	14.45
Coiflet 5	5	18.45	18.35	75.39	14.67
Beylkin	5	18.54	18.44	75.73	14.63
Daubechies 6	5	19.13	19.03	78.16	14.36
Daubechies 8	5	19.52	19.41	79.73	14.19
Daubechies 10	5	15.97	15.88	65.24	15.93
Daubechies 12	5	18.21	18.11	74.39	14.79
Haar	5	35.09	34.90	143.35	9.09

The original and reconstructed signal after a compression of 5:1 (50 coefficients) using the Haar and Daubechies10 dictionaries is presented in Figure 2 and Figure 3.

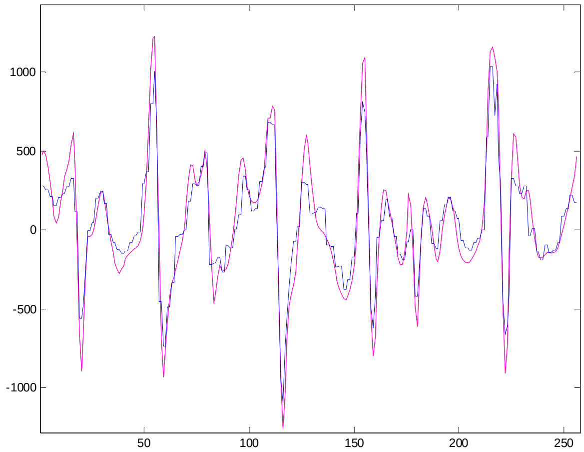

Figure 2. Original and reconstructed EEG signals for Haar wavelets (record number s030 - SNR = 9.09dB)

The poor results obtained using the Haar wavelets might be explained by the discontinuities of the Haar waves leading to a reconstructed signal having a staircase form which increases the reconstruction errors.

To evaluate the compression ratio, the original signal was represented using 12 bits and the retained coefficients using 12 bits (necessary to represent the high values of the coefficients). If a number of 50 coefficients are retained, the compression ratios are 5. In Table I, the compression results for the cases when 50 coefficients are retained.

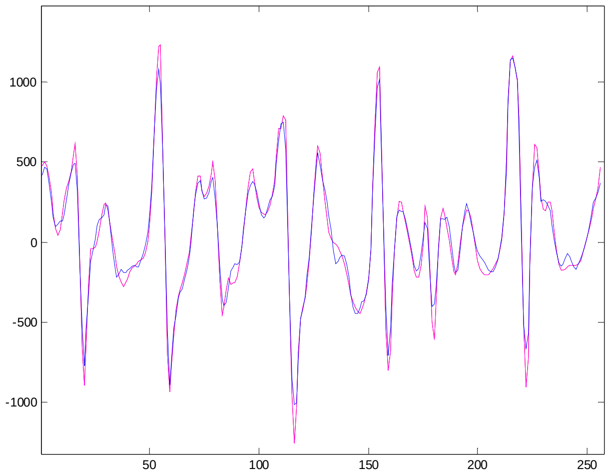

Figure 3. Original and reconstructed EEG signals for Daubechies10 wavelets (record number s030 - SNR = 15.93dB)

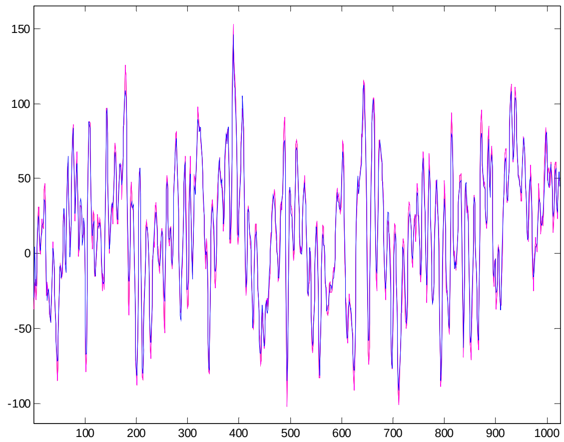

Figure 4. Original and reconstructed EEG normal signals for Daubechies10 wavelets (record number z003 - SNR = 16.9)

In Figure 4 the compression made using the Daubechies10 wavelet dictionary for a segment of normal EEG signal (1024 samples) is presented. It can be seen that the reconstruction of the EEG compressed signal is good i.e., it has a SNR = 16.9 for a CR = 5:1.

The reconstruction error for the Daubechies10 wavelet dictionary case is shown in Figure 5.

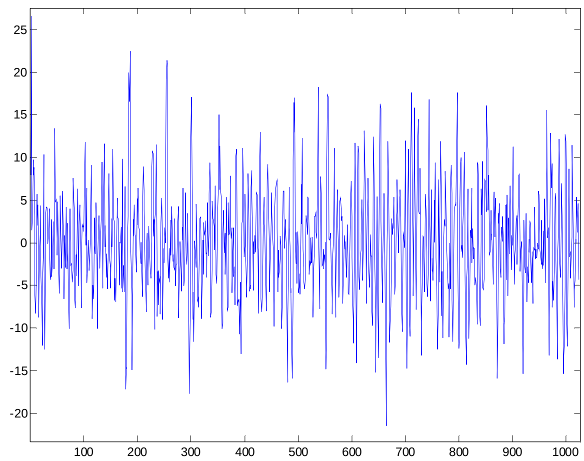

Figure 5. Reconstruction error of EEG normal signals for the Daubechies10 wavelets case

5. Experimental results and discutions for ECG signals

In order to compare the performances of various overcomplete dictionaries the record number 117 and 208 consisting of the first 256, respectively 1024 samples from the MIT-BIH Arrhythmia database have been used. The ECG signals were digitized through sampling at 360 samples/s, quantized and encoded with 11 bits.

As can be seen from Table 2, the best results from the reconstruction error point of view, were obtained using the Coiflet4 wavelet and the worst, using the Haar wavelets.

In order to compare the performances of various overcomplete dictionaries the record number 117 and 208 consisting of the first 256, respectively 1024 samples from the MIT-BIH Arrhythmia database have been used. The ECG signals were digitized through sampling at 360 samples/s, quantized and encoded with 11 bits.

The good results obtained using Coiflet4 might be explained to a certain extent by its similarity with the shape of the QRS complex.

To evaluate the compression ratio, the original signal was represented using 11 bits and the retained coefficients using 14 bits (necessary to represent the high values of the coefficients). If a number of 25 or 17 coefficients are retained, the compression ratios are 8 or 11.8 respectively.

Table 2. Results of the compression algorithm for various dictionaries when 25 and 17 coefficients were retained

	CR	PRD	PRDN	CR	PRD	PRDN
Battle 3	8	0.27	8.00	11.8	0.37	10.97
Coiflet 4	8	0.25	7.49	11.8	0.31	9.31
Daubechies 18	8	0.27	8.24	11.8	0.38	11.31
Haar	8	0.43	12.98	11.8	0.6	11.77
Symmlet 8	8	0.25	7.62	11.8	0.34	10.24

The original and reconstructed signal after a compression of 11.8:1 (17 coefficients) is presented in Figure 6 (Haar dictionaries) and Figure 7 (Coiflet4 dictionaries).

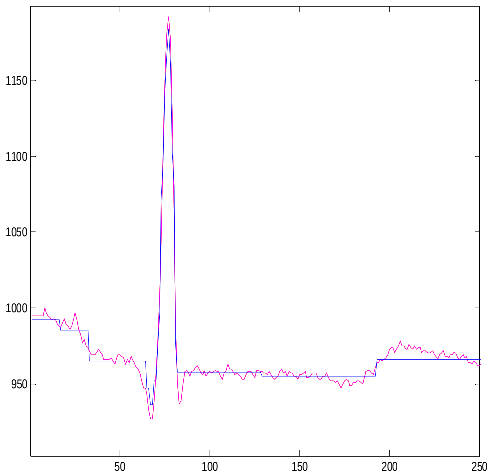

Figura 6. Original and reconstructed ECG signals for Haar wavelets (CR = 11.8)

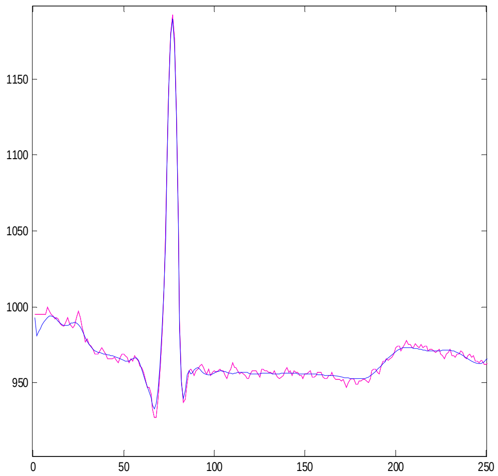

Figura 7. Original and reconstructed ECG signals for Haar wavelets (CR = 11.8)

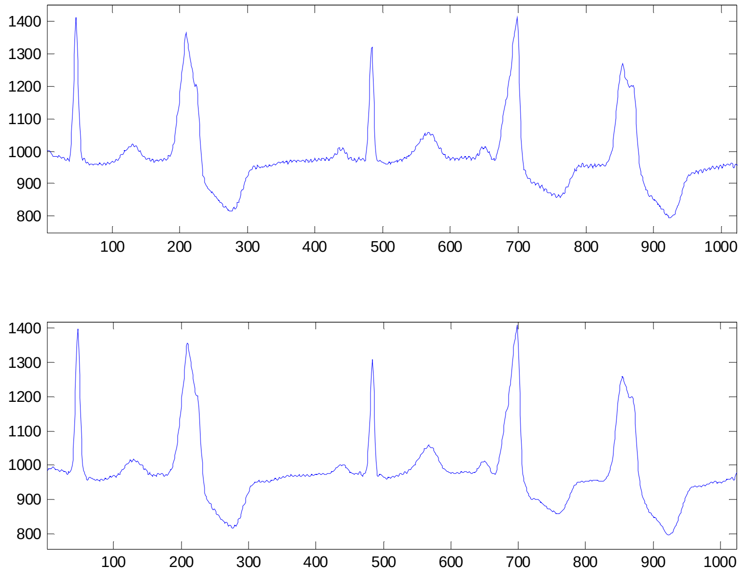

Figura 8. Original and reconstructed ECG pathologycal signals for Coiflet4 wavelets (CR = 11.8)

In Figure 8 the compression made using the Coiflet4 wavelet dictionary for a segment of pathological ECG signal (with 1024 samples) is presented. It can be seen that the reconstruction of the ECG compressed signal is good i.e., it has a PRD = 0.4 for a CR = 11.8:1 for this case.

The reconstruction error for the Coiflet4 wavelet dictionary case is shown in Figure 9.

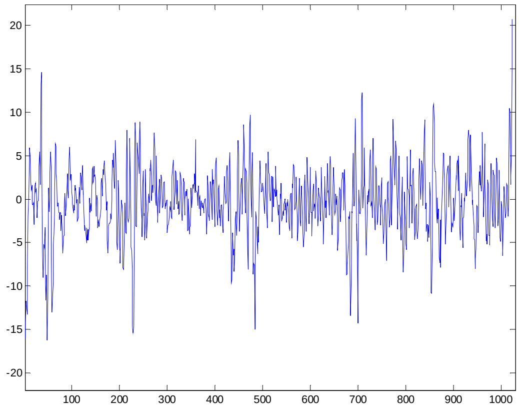

Figura 9. Reconstruction error of ECG pathologycal signals for the Coiflet4 wavelets case (CR= 11.8)

6. Discussions

The EEG and ECG signals compression results obtained with decomposition based on BP with various dictionaries have been compared with some other compression method and the results are presented in Table 3 and Table 4 [10-13].

Table 3. Comparison with other EEG lossy compression algorithms

	SNR	CR
Adaptive quantization (Hinrichs)	17dB	4 : 1
	23dB	2.6 : 1
PCA	24dB	2 : 1
	16dB	6 : 1
Fractal	20dB	6.6 : 1
Functional approximation	60dB	1.5 : 1
NN	Not report	5 : 1
BP Daubechies10	15.93dB	5 : 1

Table 4. Comparison with other ECG compression algorithms

	PRD	CR
Wavelet and Huffman	3.2	9.4:1
JPEG2000	0.86	8:1
	1.03	10:1
SPHIT	1.18	8:1
Hilton	2.6	8:1
Djohn	3.9	8:1
AZTEC	28	10:1
TP	5.3	2:1
CORTES	7	4.8:1
Fan/SAPA	4	3:1
BP Coiflet 4	0.31	11.8:1

5. Conclusion

A comparison of various overcomplete dictionaries for EEG signals compression using the basis pursuit principle has been performed. It has been found that the best approximation using the BP as a method of decomposition/compression of EEG signals corresponded to the Daubechies10 wavelet for which, for a compression ratio of 5:1, the SNR values were 15.93dB. The results compare favorably with other EEG compression methods.

For ECG signal, it has been found that the best approximation using the BP as method of decomposition/compression of ECG signals corresponds to the Coiflet 4 wavelet for which, for a compression ratio of 8 and 11.8, the PRD values were 0.25 and 0.31 respectively. Again, the results compare favorably with other ECG compression methods.

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