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Linear Quadratic State Feedback Design for Switched Linear Systems with Polytopic Uncertainties

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Abstract

In this paper, we consider the design of stable linear switched systems for polytopic uncertainties via their closed loop linear quadratic state feedback regulator. The closed loop switched systems can stabilize unstable open loop systems or stable open loop systems but in which there is no solution for a common Lyapunov matrix. For continuous time switched linear systems, we show that if there exists solution in an associated Riccati equation for the closed loop systems sharing one common Lyapunov matrix, the switched linear systems are stable. For the discrete time switched systems, we derive an LMI to calculate a common Lyapunov matrix and solution for the stable closed loop feedback systems. These closed loop linear quadratic state feedback regulators guarantee the global asymptotical stability for any switched linear systems with any switching signal sequence.

Keywords: *Continuous time linear switched system, discrete time switched linear systems, linear quadratic state feedback regulator, common Lyapunov matrix*

1. Introduction

A switched linear system is hybrid dynamical system which consists of several linear subsystems and a switching rule that decides which of switching rule is active at each moment. In the last two decades, there has been increasing interest in stability analysis and control design for switched systems in [1], [2], [3], [4], [5], [6], [7], [8], [9] and [12] The motivation for studying switched systems is from the fact that many practical systems are inherently multimodal. Many researchers have studied the use of multiple models in adaptive control of both linear and nonlinear in which controllers are switched depending on which model provides the least identification errors. Stability results for such continuous time, switching control systems have been shown for the linear [10] and for a certain nonlinear case [11]. The linear multiple model switching is similar to the control of Markovian jump linear systems or the system has models whose parameters change with respect to an underlying Markov chain.

This paper assumes that the switching signal can be designed by control engineers. Hence, we investigate the design of stable linear switched systems for polytopic uncertainties via their closed loop linear quadratic state feedback regulators. The closed loop linear switched systems can stabilize unstable systems or stable systems but there is no solution for a direct common Lyapunov matrix. It is clearly that the quadratic stability requires for uncertain systems a quadratic Lyapunov function which guarantees asymptotical stability for all uncertainties.

The outline of this paper is as follows. In section 2, we consider the linear quadratic state feedback design for continuous time case. Section 3 presents the linear quadratic state feedback design for discrete time case. Three examples are provided in section 2 and section 3 to illustrate the main ideas in each section. Finally in section 4, conclusions are drawn and some directions of future research are discussed.

2. Linear Quadratic State Feedback Design for Continuous-Time Case

In this section, we consider the continuous-time switched linear system

$$\dot{x}(t) = A_{\sigma(x,t)} x(t) \tag{2.1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $\sigma(x,t)$ is a switching rule defined by $\sigma(x,t): \mathcal{R}^n \times \mathcal{R}^+ \to \{1, 2, ..., N_i\}$, and $\mathcal{R}^+$ denotes nonnegative real numbers. Therefore, the switched system is composed of continuous time combination of

$$CS_i: \ \dot{x}(t) = A_i x(t) \ \text{ for } \ i = \{1, 2, ..., N_i\} \tag{2.2}$$

Here, we assume that CS_i are uncertain polytopic type described as:

$$A_i = \sum_{j=1}^{N_j} \mu_{ij} A_j \ \text{ for } \ j = \{1, 2, ..., N_j\} \tag{2.3}$$

where N_j are the number of the extreme points of the polytope (constant matrices) $A_j = \{A_1, A_2, ..., A_{N_j}\}$ and the weighting factors $\mu_i = \{\mu_{i1}, \mu_{i2}, ..., \mu_{iN_j}\}$ belongs to

$$\mu_i : \sum_{j=1}^{N_j} \mu_{ij} = 1, \ \mu_{ij} \geq 0 \tag{2.4}$$

As indicated in [1], even if each of matrices A_j and all switched systems CS_i are globally stable with their eigenvalues being absolutely negative, there can exist a switching sequence that destabilizes the close-loop dynamics. For all given stable matrices A_j, the stability of the switched systems CS_i is guaranteed if we can find out a common Lyapunove matrix P.

Lemma 2.1: *The switched linear systems CS_i for stable polytopic uncertainties A_j can guarantee the global asymptotical stability for any switched linear systems with any switching signal sequence if there exists a common positive symmetric definite matrix $P = P' > 0$ and positive symmetric definite matrices $Q_j = Q_j' > 0$ such that $A_j P + P A_j' = -Q_j$, $\forall j$.*

Proof: Since we assume that all matrices A_j are stable and the state update equations for the linear switched systems (2.3) are $\dot{x} = A_i x = (\sum_{j=1}^{N_j} \mu_{ij} A_j) x$. For a positive Lyapunov function $V_i(x) = x'(t) P x(t)$, we have always a negative time derivative $\dot{V}_i(x) < 0$, and the system is stable for any switched systems with any switching signal sequence:

$$\dot{V}_i(x) = \left(\sum_{j=1}^{N_j} \mu_{ij} A_j x \right)' Px + x' P \left(\sum_{j=1}^{N_j} \mu_{ij} A_j x \right) = \sum_{j=1}^{N_j} \mu_{ij} x' (A_j P + P A_j') x = \sum_{j=1}^{N_j} \mu_{ij} x' (-Q_j) x < 0 \tag{2.5}$$

The existence of a direct common Lyapunov matrix $A_j P + P A_j^{'} = -Q_j$ among stable matrices A_j and positive symmetric definite matrices $Q_j = Q_j^{'} > 0$ can be searched with quadratic stability of polytopic systems or directly solved with LMIs.

Example 2.1: Consider the switched linear systems CS_i composed of four extreme points

$$A_{11} = \begin{bmatrix} -0.2 & -0.5 \\ 0.3 & 0.1 \end{bmatrix},\ A_{12} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix},\ A_{13} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}, \text{ and } A_{14} = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix} \qquad (2.6)$$

All above matrices are stable (their eigenvaluse p_i of those matrices are absolutely negative):

$$p_{11} = -0.05 \pm 0.3571i,\ p_{12} = -1 \pm 1i,\ p_{13} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}, \text{ and } p_{14} = -2 \pm 1i \qquad (2.7)$$

Searching with quadratic stability of polytopic systems, we find out a common Lyapunov matrix for all four matrices $P_{1a} = \begin{bmatrix} 0.8928 & 0.4107 \\ 0.4107 & 1.5454 \end{bmatrix}$.

Solving directly with LMIs, we find out another similar common Lyapunov matrix $P_{1b} = \begin{bmatrix} 0.71311 & 0.2920 \\ 0.2920 & 1.0851 \end{bmatrix}$. Here, we can conclude that the switched linear systems CS_i are stable with any switched linear systems and with any switching signal sequence.

It is difficult to find out a direct common Lyapunov matrix P in (2.5) for all extreme points of the polytope A_j. In this example, if we only change A_{14} in equation (2.6) by a new a stable matrix $A_{14}^{New} = \begin{bmatrix} -0.25 & 0.5 \\ -1 & 0.1 \end{bmatrix}$ with small eigenvalues $p_{14}^{New} = -0.075 \pm 0.685i$, there is no solution for a common Lyapunov matrix for four stable matrices.

The assumption that all matrices A_j are stable seems usually unrealistic. Thus, how can we deal with unstable systems or stable systems but in which there is no solution for a common Lyapunov matrix?

For stabilizing all possible switched linear systems with any switching sequence, we investigate the design of a close loop quadratic state feedback regulator, one of the most common useful algorithms in control theory. The algorithm allows to change from unstable open loop dynamics $\dot{x}(t) = Ax(t)$ into some stable closed loop dynamics $\dot{x} = (A - BK)x = A^C x$.

In the design of a linear quadratic state feedback regulator for a continuous time system, we seek to find out the stable linear state feedback gain K in $u = -Kx$ for the state space model $\dot{x} = Ax + Bu$, where B is the controller input (matrix). The linear quadratic regulator returns the solution $P = P^{'} > 0$ of the associated Riccati equation:

$$A^{'}P + PA - (PB)R^{-1}(B^{'}P) + Q = 0 \qquad (2.8)$$

where $R = R^{'} > 0$ and $Q = Q^{'} > 0$ are weighted matrices for input and state. The regulator minimizes the quadratic cost function $J(u) = \int_0^\infty (x^{'}Qx + u^{'}Ru)dt$, then the optimal linear feedback gain K:

$$K = R^{-1}(B^{'}P) \qquad (2.9)$$

By fixing R and selecting upper hand a common Lyapunov matrix P, we can design new stable closed loop matrices $A_j^C = (A_j - B_j K_j)$ which stabilize all switched systems CS_i applied to closed loop matrices A_j^C with any switching signal sequence.

For simplicity, we set $R = I$, and the closed-loop common Lyapunov $P = I$, the equations (2.8) and (2.9) become:

$$A_j^{'} + A_j - B_j B_j^{'} + Q_j = 0 \tag{2.10}$$

and

$$K_j = B_j^{'} \tag{2.11}$$

Then, the close loop matrices are

$$A_j^C = A_j - B_j B_j^{'} \tag{2.12}$$

Lemma 2.2: *The switched linear systems CS_i for stable closed loop matrices A_j^C can guarantee the global asymptotical stability for any switched linear systems with any switching signal sequence if there exists solution for the control matrices $B_j \neq 0$ and the positive symmetric matrices $Q_j = Q_j^{'} > 0$ in the associated Riccati equation (2.10).*

Proof: As indicated in Lemma 2.1, the switched systems CS_i for stable closed loop matrices A_j^C can guarantee the global asymptotical stability if there exists a positive symmetric definite matrix (common Lyapunov matrix) $P = P^{'} > 0$ and positive symmetric definite matrices $Q_j = Q_j^{'} > 0$ such that $A_j^C P + P A_j^{C'} = -Q_j$, $\forall j$. Since we set the common Lyapunov matrix $P = I$ for all stable closed loop matrices A_j^C, the negative time derivative $\dot{V}_i(x) < 0$ then becomes $A_j^C + A_j^{C'} < 0$. From (2.12), we have:

$$A_j^{C'} + A_j^C = A_j - B_j B_j^{'} + A_j^{'} - B_j B_j^{'} = (A_j + A_j^{'}) - 2B_j B_j^{'} \tag{2.13}$$

Replace $A_j^{'} + A_j = B_j B_j^{'} - Q_j$ from equation (2.10) to equation (2.13) , we have:

$$A_j^{C'} + A_j^C = -Q_j - B_j B_j^{'} < 0 \tag{2.14}$$

So that for any positive Lyapunov function $V_i(x) = x^{'}(t) P x(t)$ of the stable closed loop matrices A_j^C, we have always a negative time derivative $\dot{V}_i(x) < 0$ in (2.14), and the system is stable with any switched linear systems and with any switching signal sequence CS_i .

Example 2.2: This example is taken in [2]. Consider a switched linear system CS_i composed of four unstable matrices:

$$A_{21} = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix},\ A_{22} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix},\ A_{23} = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}, \text{and } A_{24} = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \tag{2.15}$$

Solving the Reccati equations in (2.8) with $R = R^{'} = I$, and a common Lyapunov matrix $P = P^{'} = I$, we find out the solution for the corresponding control matrices $B_j \neq 0$ in (2.10) as:

$$B_{21} = \begin{bmatrix} -0.9557 & -0.4551 \\ -0.4551 & -0.9557 \end{bmatrix},\ B_{22} = \begin{bmatrix} -0.4804 & 0.0204 \\ 0.0204 & -0.4804 \end{bmatrix},$$
$$B_{23} = \begin{bmatrix} 0.9557 & -0.4551 \\ -0.4551 & 0.9557 \end{bmatrix}, \text{and } B_{24} = \begin{bmatrix} -0.4804 & -0.0204 \\ -0.0204 & -0.4804 \end{bmatrix} \tag{2.14}$$

and the corresponding positive symmetric matrices $Q_j = Q_j^{'} > 0$,

$$Q_{21} = \begin{bmatrix} 3.1349 & -3.1232 \\ -3.1232 & 3.1349 \end{bmatrix},\ Q_{22} = \begin{bmatrix} 2.2354 & -2.0231 \\ -2.0231 & 2.2354 \end{bmatrix},$$
$$Q_{23} = \begin{bmatrix} -0.9557 & -0.4551 \\ -0.4551 & -0.9557 \end{bmatrix}, \text{ and } Q_{24} = \begin{bmatrix} 3.1349 & 2.0231 \\ 2.0231 & 2.2354 \end{bmatrix} \quad (2.14)$$

And then, the lemma 2.2 is hold, the switched linear systems CS_i applied to stable closed loop matrices A_j^C are stable with any switched linear systems and with any switching signal sequence.

For stable open loop matrices in which we cannot find out a common Lyapunov matrix as shown in example 2.1 with $A_j = \{A_{11}, A_{12}, A_{13}, A_{14}^{New}\}$, we can also re-design new stable closed loop matrices that assure stability for any switched linear systems with any switching signal sequence of CS_i:

$$A_{11}^C = \begin{bmatrix} -0.4583 & -0.4859 \\ 0.3141 & -0.2007 \end{bmatrix},\ A_{12}^C = \begin{bmatrix} -1.2513 & -1.000 \\ 1.000 & -1.2513 \end{bmatrix},$$
$$A_{13}^C = \begin{bmatrix} -2.2509 & 0.9996 \\ 0.9996 & -2.2509 \end{bmatrix},\ A_{14}^C = \begin{bmatrix} -0.5503 & -0.5912 \\ -0.9088 & -0.3208 \end{bmatrix} \quad (2.15)$$

3. Linear Quadratic State Feedback Design for Discrete-Time Case

In this section, we consider the discrete time switched linear systems

$$x(k+1) = A_{\sigma(x,k)} x(k) \quad (3.1)$$

where $x(k) \in \mathcal{R}^n$ is the state vector, $\sigma(x,k)$ is a switching rule defined by $\sigma(x,k): \mathcal{R}^n \times \mathcal{R}^+ \to \{1, 2, ..., N_i\}$, and $\mathcal{R}^+$ denotes nonnegative integers. Therefore, the switched system is composed of discrete time combination of:

$$CS_i: \ x(k+1) = A_i x(k) \ \text{ for } i = \{1, 2, ..., N_i\} \quad (3.2)$$

Similarly, we assume that CS_i are uncertain polytopic type described as

$$A_i = \sum_{j=1}^{N_j} \mu_{ij} A_j \ \text{ for } \ j = \{1, 2, ..., N_j\} \quad (3.3)$$

where N_j are the number of the extreme points of the polytope (constant matrices) $A_j = \{A_1, A_2, ..., A_{N_j}\}$ and the weighting factors $\mu_i = \{\mu_{i1}, \mu_{i2}, ..., \mu_{iN_j}\}$ belongs to

$$\mu_i: \sum_{j=1}^{N_j} \mu_{ij} = 1,\ \mu_{ij} \ge 0 \quad (3.4)$$

As indicated in [1], even if each of matrices A_j and all switched systems CS_i are globally stable with their eigenvalues $0 < p_{ii} < 1$, there can exist a switching sequence that destabilizes the close-loop dynamics. For all given stable matrices A_j, the stability of the switched systems CS_i is guaranteed if we can find out a common Lyapunove matrix P.

Lemma 3.1: *The switched linear systems CS_i for stable polytopic uncertainties A_j can guarantee the global asymptotical stability for any switched linear systems with any switching* *signal sequence if there exists a common positive symmetric definite matrix* $P = P' > 0$ *and a scalar* $\gamma > 0$ *such that* $\begin{bmatrix} P & PA_j' & \gamma \\ A_j P & P & 0 \\ \gamma & 0 & \gamma I \end{bmatrix} > 0, \ \forall j$.

Proof: Since we assume that all matrices A_j are stable and the state update equations for the linear switched systems (3.3) are $x(k+1) = A_i x(k) = (\sum_{j=1}^{N_j} \mu_{ij} A_j) x(k)$. For the stable discrete time systems, we always have the Lyapunov function decreasing $V_i(k) = x'(k)Px(k)$ and $V_i(k+1) - V_i(k) < 0$, and the system is stable for any switched systems with any switching signal sequence since they share a common Lyapunov matrix $P = P' > 0$:

$$V_i(k+1) - V_i(k) = \left(\sum_{j=1}^{N_j} \mu_{ij} A_j x \right)' P \left(\sum_{j=1}^{N_j} \mu_{ij} A_j x \right) - xPx < 0 \rightarrow A_j P A_j' - P < 0 \tag{3.5}$$

By adding a scalar $\gamma > 0$ in equation (3.5), we have $P - A_i' P A_i - \gamma I > 0$, or $P - (A_i' P) P^{-1} (P A_i) - (\gamma) I \gamma^{-1} (\gamma) > 0$. And using Schur complement, this equation is equivalent to the LMI in lemma 3.1.
Hence, the common Lyapunov matrix in lemma 3.1 is the solution to the following LMI: $\min_{P>0,\ \gamma>0} \gamma$, subject to $\begin{bmatrix} P & PA_j' & \gamma \\ A_j P & P & 0 \\ \gamma & 0 & \gamma I \end{bmatrix} > 0, \ \forall j$.

The assumption that all matrices A_j are stable (their eigenvalues $0 < p_{ii} < 1$) seems usually unrealistic. How can we deal with unstable discrete systems or stable discrete systems but in which there is not a common Lyapunov matrix?.

For stabilizing all switched linear systems with any switching signal sequence, we investigate the design of a closed loop quadratic feedback regulator for these unstable matrices. The regulator computes the optimal input that minimizes the objective function in quadratic form at each sampling time $J(k) = \sum_{i=0}^{\infty} \{ x(k+i) Q x(k+i) + u(k+i) R u(k+i) \}$ by a linear feedback control law $u(k+1) = Ku(k)$ for the closed loop stable state feedback $x(k+1) = A_j x(k) + B_j u(k)$, in which Q and R are symmetric positive weighting matrices. The regulator allows to change from unstable open loop update $x(k+1) = A_j x(k)$ into some stable closed loop update $x(k+1) = (A_j - B_j K_j) x(k) = A_j^C x(k)$.

Lemma 3.2: *The switched linear systems* CS_i *for stable closed loop matrices* A_j^C *can guarantee the global asymptotical stability for any switched linear systems with any switching signal sequence if there exists solution for a common positive symmetric Lyapunov matrix*

$P = P^{'} = P_L^{-1}$, *positive symmetric matrices* $Q = Q^{'} > 0$, $R = R^{'} > 0$, *and matrice* $K_j = Y_j^{-1} \neq 0$ *and* $B_j \neq 0$ *satisfying an LMI* $\begin{bmatrix} -R & Y_j^{'} & (A_j Y_j - B_j)^{'} & (Y_j Q^{1/2})^{'} \\ Y_j & -P_L & 0 & 0 \\ (A_j Y_j - B_j) & 0 & P_L & 0 \\ (Y_j Q^{1/2}) & 0 & 0 & I \end{bmatrix} > 0$, $\forall j$.

Proof: Suppose there exists a Lyapunov function $V(x)$ with $V(0) = 0$ and $V(x(k)) = x(k)^{'} Px(k)$ with Lyapunov matrix $P = P^{'} > 0$. The system will be stable if the Lyapunov function is decreasing, that is, $V(x(k+1)) - V(x(k)) < 0$. Suppose

$$V_i(k+1) - V_i(k) = x(k+1)^{'} Px(k+1) - x(k)Px(k) \leq -x(k)Qx(k) - u(k)Ru(k), \ \forall k \tag{3.6}$$

Then, we have $P - (A_j - B_j K_j)^{'} P(A_j - B_j K_j) - Q - K_j^{'} R K_j > 0$. Set $P_L = P^{-1}$ and $K_j = Y_j^{-1}$, the above equation can be transformed as

$$(-R) - Y_j^{'}(-P_L)^{-1} Y_j - (A_j Y_j - B_j)^{'} (P_L)^{-1} (A_j Y_j - B_j) - (Y_j Q^{1/2})^{'} (I)^{-1} (Y_j Q^{1/2}) > 0 \tag{3.7}$$

Using the Schur complement, equation 3.7 is equivalent to the LMI in lemma 3.2. The system becomes stable for any switched linear systems CS_i and with any switching signal sequence since all stable closed loop state feedback $A_j^C = (A_j + B_j K_j)$ share one common Lyapunov matrix, the solution of the LMI in lemma 3.2.

Example 3.2: This example is taken in [12]. Consider the following set of uncertain discrete models with model 1: $x(k+1) = \begin{bmatrix} 0.90 & -0.80 \\ 0.30 & 0.80 \end{bmatrix} x(k) + \begin{bmatrix} 0.20 \\ 0.20 \end{bmatrix} u(k)$ and model 2: $x(k+1) = \begin{bmatrix} 0.85 & -0.75 \\ 0.35 & 0.75 \end{bmatrix} x(k) + \begin{bmatrix} 0.10 \\ 0.10 \end{bmatrix} u(k)$. Applied the linear quadratic feedback as described in [12] with the weighted matrices as stated in lemma 3.2: $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $R = [1]$. Solved directly with LMIs, we can find out a common Lyapunov mantrix for this set: $P = \begin{bmatrix} 3.3611 & 0.2859 \\ 0.2859 & 1.2163 \end{bmatrix} > 0$. Then this closed loop hybrid system is global asymptotical stable for any switched linear systems with any switching signal sequence.

4. Conclusions

In this paper, we have considered the replacement of unstable linear switched systems for polytopic uncertainties via their closed loop linear quadratic state feedback regulator. For both continuous time and discrete time linear switched systems, if there exists solution for the closed loop state feedback, the switched linear systems always guarantee the global asymptotical stability for any switched linear systems with any switching signal sequence.

There are several important issues which should be studied in the future work. First, we set the closed loop common Lyapunov matrix $P = I$ in equation (2.10) and find solution of input matrices B_j for the stable closed loop matrices $A_j^C = A_j - B_j B_j^{'}$. Then, problems are still open for general case to find both variable matrices P and R_j. For discrete time switched linear systems, the stabilizability condition (3.7) is directly derived from the

Lyapunov function (3.6). We can also solve the solution for the state feedback gain K and the Lyapunov matrix P via the associated discrete time Riccati equation using linear quadratic regulator design for discrete time systems. Feasible solution of the state feedback design with sufficient conditions that guarantees the parameter dependent Lyapunov function is also needed further studies.

References

[1] Branicky, M., "Multiple Lyapunov Functions and Other Analysis Tools for Switched and Hybrid System". *IEEE Trans. Auto. Contr*. , 1998, Vol. 43, No. 4, pp. 475-482.
[2] Zhai, Guisheng, Hai Lin, and Pasos Antsaklis. "Quadratic Stabilizability of Switched Linear Systems with Polytopic Uncertaities". *Int. J. Control*, 2003, Vol. 76, pp 747-753.
[3] Raymond A., Michael S, Stefan P., and Bengt L. "Perspectives and Results on the Stability and Stabilization of Hybrid Systems". Proc IEEE, 2000, Vol. 88, pp. 1069-1082.
[4] Liberson D. "Stabilizing a Linear System with Finite State Hybrid Output Feedback". Proc of MED99, 1999, pp. 176-183.
[5] Vesely V. "Static Output Feedback Robust Controller Design via LMI Approach" Journal of Electrical Engineering, 2005, Vol. 56, No. 1-2.
[6] Akar, M., Paul, A. Michael, G., Mitra, U. "Conditions n the Stability of a Class of Second-Order Switched Systems". IEEE Tran on Aut Col., 2006, Vol 51(2), pp. 338-340.
[7] Berg, R., and Pogromsky, J. "Convergent Design of Switched Linear Systems". Proc. IFAC Conf on Analysis and Design of Hybrid Systems, Italy, 2006, Vol. 2, pp 122-128.
[8] Kim K, and Park Y. "Sliding Mode Design via Quadratic Performance Optimization with Pole-Clustering Constraint" SIAM J. Cont. Optim., 2004, Vol 43(2), pp. 670-684.
[9] Jianbo, L. and Robert E. "Optimal Hybrid Control for Structures". Computer-Aided Civil and Infrastructure Engineering, 1998, Vol. 13, pp. 405-414.
[10] Narendra, K. and Balakrishnan, J. "Adaptive Control Using Multiple Models". IEEE Trans on Automatic Control, 1997, Vol 42(2), pp. 171-187.
[11] Narendra K, and Xiang, C. "Adaptive Control of Simple Nonlinear Systems using Multiple Models". Proc. of American Cont. Conf. , 2002, A.K, pp. 1779-1784.
[12] Vu Trieu Minh and Nitin Afzulpurkar. "Robust Model Predictive Control for Input Saturated and Soften State Constraints". *Asian Journal of Control*, 2005, Vol. 7, No. 3. pp. 323-329.

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