Approximation Studies on Image Enhancement Using Fuzzy Technique

Sasi Gopalan[1], Madhu S Nair[2] and Souriar Sebastian[3]
[1]*School of Engineering,*
Cochin University of Science and Technology (CUSAT),
Cochin – 682 022, Kerala, India.
{ sasigopalan@cusat.ac.in}
[2]*Rajagiri College of Social Sciences, Kochi, Kerala, India*
{madhu_s_nair2001@yahoo.com}
[3] *St. Alberts College, Kochi, Kerala, India*

Abstract

An image can be considered as a fuzzy subset of plane. Fuzzy entropy measuring the blur in an image is a functional which increases when the sharpness of its argument image decreases. In this paper we consider three parameters such as the intensification parameter t, the fuzzifier f_h *and crossover point* μ_c*.The partially ordered set and lattice theory defined in section 2 strengthens the selection of any value in the interval [0, 1] such that the image can be enhanced. Here we verify the approximate value of* μ_c *and visual observation image enhancement in different figures. By entropy minimization technique we construct an approximation of the ideal image and for that we apply the enhancement operation to the low contrast image. This can be done by an optimization process that minimizes a criterion function.*

Keywords: *Fuzzy sets, Approximation, Lattice, Partially ordered set, Cross over point, Lagrange multipliers, Tenengrad measure*

1. Introduction

The ultimate aim of image processing is to use data contained in the image to enable the system to understand, recognize and interpret the processed information available from the image pattern. The fuzzy set theory is incorporated to handle uncertainties (arising from deficiencies of information available from situation like the darkness may result from incomplete, imprecise, ill-defined, and not fully reliable, Vague, contradictory information) in various stages of a pattern recognition system. The fuzzy logic provides a mathematical framework for representation and processing of expert knowledge. The concept of if-then rules play a role in approximation of the variables like cross over point. Also the uncertainties within image processing tasks are not always due to randomness but often due to vagueness and ambiguity. Fuzzy techniques enables us to manage these problems effectively. A gray tone image process ambiguity within pixels (points) due to possible multi-valued levels of brightness in the image. Incertitude in an image pattern explained in terms of grayness ambiguity or spatial (geometrical) ambiguity or both. Grayness ambiguity means indefiniteness in deciding whether a pixel is white or black. Spatial ambiguity refers to indefiniteness in the shape and geometry of region within the image. In this paper we deal with spatial ambiguity. Since the regions in an image are not always defined, uncertainty can arise within every phase. Basic principles of image processing and recognition using fuzzy set theory is defined in [3].In the light of fuzzy set theory different aspects of image processing and analysis is discussed in [19] . Based on [3] the relations between objects with indeterminate boundaries are called approximate topological relations since they constitute an approximation of classical topological relations between objects with crisp boundaries.

Rule based fuzzy systems are designed in order to apply the principles of approximate reasoning to digital image processing. This paper shows the use of fuzzy sets in image enhancement and its approximation in the fuzzy domain, so that it can be seen that fuzzy systems produce a sequence of improved approximations on the entropy measure (Measure fuzziness) with respect to the crossover point, leading to image enhancement. Because of poor and non uniform lighting conditions of the object and the non linearity of the imaging system, vagueness is introduced in the acquired image. This vagueness in the image appears in the form of imprecise boundaries and color values during image digitization.

Freeman J (1975) introduced in [4] approximate (fuzzy) nature of special relations among spatial objects and suggested that these special solutions be described in an approximated framework. The studies in [5] explained about the potential fuzzy set theory in image processing. Some authors like Rosen field [6] explained the concept of fuzzy geometry of a gray image which is a generalization of many crisp properties and relations among regions in an image. These extensions include connectedness, adjacency and sharpness, convexity, area, perimeter, compactness, width, etc. Fuzzy entropy is a functional on fuzzy sets that becomes smaller when the contrast of its argument fuzzy set is improved. Fuzzy entropy can also be considered as a measure of confusion between object and background.

In order to separate an object from background in a noisy image one often tries to approximate the object by suitable fitting function. This can be done different type of relaxation methods aiming at minimizing some measure of mismatch between the image at one hand and the real unknown image before degradation. In our discussion Shannon entropy is converted into fuzzy entropy that can be used in the image enhancement process. Since fuzzy entropy is a measure of the amount of fuzziness in an image we can use the global contrast function J_c in a minimization process. Using the results defined in [14, 15] we explained about the basic concepts of the fuzzy approximation in image enhancement.

Definition1.1

Fuzzy set. A fuzzy set in a universe of discourse U is characterized by a membership function $\mu_A : U \to [0,1]$. A fuzzy set is viewed as the generalization of an ordinary set whose membership function takes only two values (0,1). Therefore a fuzzy set $A \in U$ may be represented as a set of ordered pairs of x and its grades membership function $A = \{x, \mu_A(x) / x \in U\}$ When U is discrete the fuzzy set A can be represented as $A = \sum_{i=1}^{n} \mu_A(x_i)/(x_i)$.

Definition1.2

Cross over point: The element $x \in U$ at which $\mu_A(x) = 0.5$ is called Cross over point.

Definition1.3

A fuzzy set whose support is a single point in U with $\mu_A(x) = 1$ is referred to as fuzzy singleton.

2. Use of fuzzy sets in image processing and approximation studies

In the field of image enhancement, a rule is capable of performing a simple smoothing action as follows: If a pixel is much brighter (darker) than neighboring pixels THEN reduce (increase) its luminance, else leave it unchanged. Application of theory of fuzzy sets to image analysis is to consider images as fuzzy subsets of a plane. The use of fuzzy sets provides a basis for a systematic way for the implementation of vague and imprecise concepts. The manipulation of these concepts leads to theory of approximation using fuzzy systems in image processing. If the observed data are disturbed by random noise then the fuzzification operator should convert the probabilistic data into fuzzy numbers or fuzzy (possibilistic) data, so that computational efficiency is enhanced since fuzzy numbers are much easier to manipulate than random variables.

Some authors like Dubois and Prade [18] defined a bijective transformation which transforms a probability measure into possibility measure by using the concept of the degree of necessity. Based on this method the histogram of the measured data may be used to estimate the membership function for the transformation of probability into possibility. Recent studies explained that histogram is used as the basis for fuzzy modeling of color images. Here histogram is used to convert the Shanon entropy to the fuzzy entropy. Fuzzy image processing is a kind of nonlinear and knowledge based image processing. Here the image fuzzification means generation of suitable membership values. A topological relationship such as connectedness and adjacency is defined for binary images. The extension of digital topology to fuzzy sets [6] builds the necessary frame work for many fuzzy logical operations.

The concept of fuzziness measure or entropy measures means the correspondence between approximate evaluation mappings corresponding to the improved contrast relation. Using entropy minimization technique one constructs an approximation of the ideal image of which the image at hand is a degraded version. The enhancement of image which is a crucial step is usually implemented as an optimization process that minimizes a criterion function. In this discussion, fuzzy entropy is used as a global contrast functional to force the fitting function to be as close as possible to the original.

Definition 2.1

The notion of a partial order is of fundamental importance of binary relation $\leq$ between elements of P is called partial order if it satisfies the following conditions $\forall x, y, z \in P$

1.Reflexivity: $x \leq x$

2.Anti Symmetry: $(x \leq y$ and $y \leq x) \Rightarrow x = y$

3.Transitivity: $(x \leq y$ and $y \leq z) \Rightarrow x = z$

Proposition 2.1

If $\leq$ is a partial order on P then $(P, \leq)$ is called a partially ordered set or equivalently a poset .A lattice is a poset $(L, \leq)$ with the additional property that any two of its elements have greatest lower bound (g.l.b) and a least upper bound(l.u.b).Given a lattice $(L, \leq)$, then for any two elements $x, y \in L$ the g.l.b is the meet of x and y and is denoted by $x \wedge y$ then l.u.b is called the join of x and y and is denoted by $x \vee y$. Lattice elements of x and y are comparable when one of the relations $x \leq y$, $y \leq x$ holds. Otherwise x and y are in comparable. A lattice with in comparable elements is called ordered lattice. For example, the set R of real numbers is a totally ordered lattice.

Proposition: 2.2

For a given poset (lattice) an additional poset can be derived as: Let $\leq$ be partial order on a set L. Define the relation $\leq$ on the set $L \times L$ as $(x, y) \leq (z, u) \Leftrightarrow x \leq z$ and $y \leq u$ then $\leq$ is a partial order on $L \times L$. If $(L, \leq)$ is a lattice then $(L \times L, \leq)$ is also a lattice.

Definition: 2.2

A metric distance in a set is a non-negative real valued function $d : S \times S \rightarrow R$ which satisfies:

d_1. $d(x, x) = 0$

d_2. $d(x, y) = 0 \Rightarrow x = y$

d_3. $d(x, y) = d(y, x)$

d_4. $d(x, y) \leq d(x, z) + d(z, y)$ .If d_1, d_2, d_3 are satisfied then d is called pseduometric.

Definition2.3

A Valuation in a lattice $(L, \leq)$ is a function $v : L \rightarrow R$, if $\forall x, y \in L$ satisfies $v(x) + v(y) = v(x \wedge y) + v(x \vee y)$.

Definition2.4

A valuation is said to be positive if $\forall x, y \in L$, $x \prec y \Rightarrow v(x) \prec v(y)$.

Definition2.5

A valuation is said to be isotone if and only if $x \leq y \Rightarrow v(x) \leq v(y)$

Proposition2.3

Let $(L, \leq)$ be a lattice and v be a positive valuation then $d(x, y) = v(x \vee y) - v(x \wedge y)$ is a metric distance.

Definition2.6

Let X is a set and I= [0, 1] be unit interval of the real line. Define a fuzzy subset A of X as $A = \{(x, f(x)) / x \in X\}$ where f(x) is the degree of membership of element $x \in A$ .So if $f(x) = 1, x \in A$ and if $f(x) = 0, x \notin A$. Thus for $0 \prec f(x) \prec 1$ is a measure of degree of likelihood of x belonging to A. In our discussion X is considered as the spatial domain of a digital image. This means X is a set of pixel intensity values.. It is assumed that the fuzzy set represents an object A with gray level adjusted to the range between 0 and 1. This image can be considered as a fuzzy set $f : X \to [0,1]$.

Proposition2.4

Let F(x) denotes set of all fuzzy sets $f : X \to [0,1]$. Then F(x) can be considered as the set of all possible digital images over X. $f, g \in F(x)$ can be ordered as $f \leq g$ iff $f(x) \leq g(x) \forall x \in X$. The usual order relation in I induces the ordering in F(x). So this interval (I) turn out to be a lattice or every couple of points in the interval I have least upper bound and greatest lower bound, and so I is carried out to F(x). In [8, 20] a new ordering is defined and it describes that the unit interval are ranked according to their closeness to the midpoint of the interval.

Theorem2.1

Let $F : X \to [0,1]$ be the set of all real valued function on X. Then $f, g \in F$ with $f \leq g$, then g is more fuzziness than f.

Proof :

Consider $a_1, a_2 \in I$ with $a_1 \leq a_2$ then by proposition 2.3, $a_1 \leq a_2 \leq \frac{1}{2}$ or $\frac{1}{2} \leq a_2 \leq a_1$. According to the definitions of lattice theory, consider the functions $f, g \in F(x) \subset X$ with $f \leq g$ .Then $f(x) \leq g(x) \forall x$ .So $\leq$ is a partial order relation. Since fuzzy sets are function with co-domain I, it turns out to F(x). So $f \leq g$ iff $f(x) \leq g(x) \leq \frac{1}{2}$ or $\frac{1}{2} \leq g(x) \leq f(x)$. This means that $g(x)$ is with more fuzziness than $f(x)$, since it is close to the midvalue1/2.

Proposition2.5

If $f(x)$ is the spatial domain of two images f and g then $f(x)$ and $g(x)$ are respective gray level values at a point $x \in X$ then by proposition 2.1, g is less contrast than f.

Proposition 2.6

From [7, 8], entropy is a real valued function E on F(x) such that E (f) =0 iff f is crisp and E (f) is maximum iff E (f) is maximally fuzzy. $E(f) \leq E(g)$ Whenever f is of higher contrast than g .

Proposition2.7

$(f \wedge g)(x) = \min\{f(x), g(x)\}$ and $(f \vee g)(x) = \max\{f(x), g(x)\}$.so for every element x in the spatial domain, f and g on F(x) form a lattice.

Proposition 2.8

The partially ordered set $(F(x),\leq)$ is called an approximation evaluation set and the mapping f , g is called approximate evaluation mapping.

Definition 2.7

(Minimal and maximal elements)
Let $(Q,\leq)$ be a quasi ordered set .Then the set of minimal elements of Q is defined as $\min(Q,\leq)=\{x / x \in Q\}$ and $\forall y \in Q, y \leq x \Rightarrow x \leq y$ and the maximal elements of Q $\max(Q,\leq)=\{x / x \in Q\}$ and $\forall y \in Q, x \leq y \Rightarrow y \leq x$

Remark2.1: For a partially ordered set $(P,\leq)$ the definitions of the minimal and maximal elements reduces to

i) $\{x / x \in p\}$ and $\forall y \in P, y \leq x \Rightarrow y = x$

ii. $\{x / x \in P\}$ and $y \in P, x \leq y \Rightarrow x = y$ (These two are the generalization of quasi ordered set)

Definition2.8

It is clear from [8], X is isomorphic to I_a if there is a map $f : X \rightarrow I_a$ which is one to one, onto and both f and f^{-1} are order preserving. That is, $x \leq y$ iff $f(x) \leq f(y)$.

Proposition 2.8

Using the results shown in [7, 8] it is possible to shrink or stretch the order relation $\leq$ in various ways in the interval [0, 1].So corresponding to the constructed image a new relation $\underset{a}{\leq}$ is formed. So the mapping $f : X \rightarrow I_a$ isomorphic. Thus the set of all maps $F_a(x)$ is equipped with order relation $\underset{a}{\leq}$ according to $f \leq g$ iff $f(x) \underset{a}{\leq} g(x)$ $\forall x \in X$.

Remark 2.2: Valuations on lattices are measure and probability functions on fields of sets and it guarantees the metric on X.

Theorem 2.2

Consider the mapping $f : X \rightarrow I$. Then for each element $a \in [0,1]$ such that $f : X \rightarrow I_a$ is isomorphic and $(F(x),\underset{a}{\leq})$ is an order relation.

Proof:

By proposition 2.7 one selects any value between 0 and ½ for under exposed images and any value between ½ and 1 for over exposed images. From [7, 8], one can easily verify that the mapping $f : X \rightarrow I_a$ is isomorphic. Proposition 2.9, $\underset{a}{\leq}$ is an order relation such $(F(x), \underset{a}{\leq})$ is quasi ordered set. This shows that any value other than 0.5 can be choose for μ_c

Remark 2.3: According to proposition 2.7 and theorem 2.2 , one selects any value between 0 and ½ for under exposed images and any value between ½ and 1 for over exposed images. In our discussion the image is enhanced for under exposed images. One can easily verify that there exist one smallest element and greatest element (0 and ½) such that the smallest element is minimal and the greatest element is maximal. According to [10] the entropy measure can be seen as evaluation mappings or as approximate solutions of this evaluation problem satisfying some boundary conditions.

3. Image Enhancement

Images can be processed by optical, photographic and electronic means, but image processing using digital computers is the most common method because digital methods are fast, flexible and precise. Image enhancement improves the quality (clarity) of images for human viewing. Removing blurring and noise, increasing contrast, revealing details are examples of enhancement operations. For example consider the images given in this paper which might be of low contrast and some what blurred and extending contrast range could enhance the image. The original image might have areas of very high and very low intensity, The aim of image enhancement is to improve interpretability or perception of information in images for human viewers., or to provide better input for other automated image processing techniques. Image enhancement technique is divided into two (i). Spatial domain methods which operate on pixels and (ii) Frequency domain method which operate on the Fourier transform of an image. In this paper we have used spatial domain method.

3.1 Image Enhancement as an Inverse Problem

In an image enhancement problem defined in [14, 15], we assume that an ideal image f has been corrupted to create the measured image g. Here g is a low contrast image where, $g = [g_{1,} g_2, ... g_N]$ is the spatial intensity values and g_i denote the i^{th} intensity level in a column vector representation of the image g . The enhancement problem is the problem of finding the best estimate of f given the measurement g .Thus we have a process which takes an input and produces an output and we wish to infer the output. In this paper we have $g = [g_{0,} g_1, ... g_{255}]^T$ where pixels number is $g_{0,} g_1, ... g_{255}$ .and the corresponding frequencies are $h = [h_{0,} h_1, ... h_{255}]^T$ such that the probability $p(x)$ can be calculated. By using t, μ_c, f_h we can define the membership function μ_X such that the reconstructed image is $\mu'_X(x)$ .Then the functional E define a mapping from $[g_{0,} g_1, ... g_{255}]^T \rightarrow [g'_0, g'_1, ... g'_{255}]^T$. That is, $g' = E(g)$. Here $E(g)$ indicates entropy based optimization with respect to t, μ_c, f_h ,which is an iterative approach. Using optimum t, μ_c, f_h we can calculate $\mu'_X(x)$. From $\mu'_X(x)$ we get the approximated value of g' using inverse operation which may be close to f .

Definition: 3.1

A problem $g^{'} = Eg$ is said to be well-posed if

(i).For each g a solution $g^{'}$ exists.

(ii) The solution is unique.

(iii).The solution g is continuously depends on the data $g^{'}$.

Proposition: 3.1

If the conditions in the definition 3.1 do not hold, the problem is said to be ill posed**.** Ill-posedness is normally caused by the ill conditioning of the problem. Conditioning of the mathematical problem is measured by the sensitivity of the output to changes in the input.

Proposition: 3.2

For an ill-conditioned problem, a small change of input can change the output a great deal. According to [14], in many applications of linear algebra the need arises to find a good approximation $g^{'}$ to the function $g \in R^{n}$ satisfying an approximate equation $g^{'} = Eg$. It will be ill-conditioned or singular, when $E \in R^{m \times n}$ given by $g \in R^{m}$ where g is the results of measurements obtained by small errors .Frequently, ill-condition also arise in the iterative solution of optimization problems.

Proposition: 3.3

When $g^{'}$ is a reasonable approximation to f with $Eg = g^{'}$, then the usual error estimates $\left\| g - g^{'} \right\| \leq \left\| E^{-1} \right\| \left\| E\, g - g^{'} \right\|$. Generally any regularization method tries to analyze a treated well-posed problem whose solution approximates the original ill-posed problem. For example, in the first approach one might think of is to produce an image estimate which has the minimum least square error. That is finding the unknown image f which minimizes $J_{c} = \left\| g^{'} - Eg \right\|^{2}$. Directly minimizing J_{c} does not work as the problem is ill conditioned. In order to give preference to a particular solution with desirable properties, the regularization term $\lambda(C_{f} - C_{d})$, where C_{f} is fuzzy average contrast and C_{d} is the visual quality for under exposed images (here taken as 0.4) and λ (assumed 0.1) is the Lagrange multiplier included in this minimization processes. Regularization term is needed to derive a solution to an ill posed problem.

Corollary

The theories discussed in section 3 leads to the following theorem.

Theorem 3.1

Let f be an ideal image and g be the corrupted image from f where f and g are defined on X.Then the entropy E as the functional is mapping from $[g_{0,}g_1,...g_{255}]^T \rightarrow [g_0^{'}, g_1^{'},....g_{255}^{'}]^T$ .Then $Eg = g^{'}$ approximates f .

Proof is Trivial.

4. Transformation of fuzzy sets for Image Enhancement

A functional definition expresses the membership function of a fuzzy set in a functional form typically a bell shaped function, triangular shaped functions, trapezoidal shaped functions etc. Such functions are used in Fuzzy Logic Controllers, because they lead themselves to manipulation through the use of fuzzy arithmetic. The functional definition makes changes in the normalization of universe. Consider the functional definition

$$\mu_X(x) = e^{\left[-(x_{\max}-x)^2/2f_h^2\right]}$$

To apply the concept of fuzzy sets to image enhancement problems a measured image is essential and its histogram is used to fuzzify the images and their features. Finally it is an extension of digital topology which plays a key role in image representation and local operations. An image X of size $M \times N$ and L levels can be considered as array of fuzzy singletons (a fuzzy set with only one supporting point), each having a value of membership denoting its degree of brightness relative to some brightness level n = 0,1,…….,L-1, in the notion of fuzzy sets. Therefore we write $X = \{\mu_X(p)/p, p \in X\}$, where $\mu_X(p), 0 \le \mu_X(p) \le 1$denotes the grade of membership possessing some property μ (e.g., brightness, edge, smoothness) or belonging to some subset (e.g. object, skelton or contour) by a pixel or a point P. A fuzzy subset of an image X is a mapping $\mu : X \rightarrow [0,1]$. For any point $p \in X, \mu(p)$ is called the degree of membership p in μ (An ordinary subset X can be considered as a fuzzy subset for which μ takes only the values 0 and 1).

The global or local information of an image is used in defining a membership function, characterizing some property. For the transformation of intensity values X in the range (0-255) to the fuzzy domain a membership function of Gaussian type is used such as $\mu_X(x) = e^{\left[-(x_{\max}-\mu_c)^2/2f_h^2\right]}$. After the optimization process (iterative process), the optimum values of t, μ_c, f_h will be obtained. Then the modified membership values $\mu_X^{'}$ can be obtained by using t, μ_c, f_h. The modified membership values for each intensity level can then be transformed into spatial domain by $x^{'} = x_{\mathbf{max}} - [-2\ln(\mu_X^{'}(x))f_h^2]^{1/2}$ [1]. It is verified with various figures by visual observation and with qualitative values like Tenengrad measure. Also from the table it is clear that if the normalized universe is changed, the parameters t, μ_c, f_h changes accordingly. The choice of membership function is based on the subjective criteria of the decision. Suppose the measurable data is disturbed by noise, the membership function should be sufficiently wide to reduce the sensitivity to noise. If we use RGB color space, the operation has to be applied on three components R,G and B respectively. This will consume a lot of time and will introduce color artifacts. In such a situation HSV color space is more suitable, since the enhancement operation need to be applied only on V component, which will also lead to color preservation. The choice of grades of membership is based on the subjective criteria of the decision. The Gaussian membership function is used to model the intensity of the color contrast V property of the image, as HSV color space. This is suitable for under exposed images. The fuzzy based approaches provides automatic image contrast enhancement.

The original optimal hyper plane algorithm proposed by Vladimir Vapnik in 1963 was linear classifier. However in 1992, Bernhard Boser Isabelle Guyon and Vapnik suggested a way to create nonlinear classifiers by applying the kernel trick to maximum-margin hyper planes. If the kernel is used as a Gaussian radial basis function the corresponding feature space is a Hilbert space of infinite dimension. Maximum margin classifiers are well regularized so that infinite dimension does not spoil the results. Here the kernel with Gaussian Radial basis function is $\mu_X(x) = k(x, x') = e^{\left[-(x_{\max} - x)^2 / 2f_h^2\right]}$.

Definition 4.1

Metric entropy or ε-entropy has been introduced by Kolmogrov in order to classify compact metric sets according to their massivity.

Let A be a subset of a metric space X and $\varepsilon \succ 0$ be given. A family $U_1, \ldots\ldots U_n$ of subsets of X is an ε- covering of A if the diameter of each U_k does not exceed 2ε and if the sets U_k cover A. For a given $\varepsilon \succ 0$ the number n depends upon the covering family, but $N_\varepsilon(A) = \min n$ is the invariant of the set A.

Definition: 4.2

Entropy on F(x) is a real valued function E satisfying,

(1). (i). $E(f) = 0$ iff $\forall x \in X, f(x) = 0$.

(ii). $E(f) = 1$ iff $\forall x \in X, f(x) = 1$.

(2) $E(f)$ is maximum iff $\forall x \in X, f(x) = 0.5$.

(3) If $f \le g$ then $E(f) \le E(g)$,whenever f is sharper than g.

Definition4.3

Shanon entropy: High entropy values are indicative of disordered states and low entropy values are characteristic of ordered states. This indication of entropy is called Shannon's entropy. Generally the Shannon's function is defined as $E = -x \ln x - (1-x)\ln(1-x)$,

According to proposition 2.6 one can define the additive entropy on $F_a(x)$ with the partial order relation $\underset{a}{\le}$. Therefore the additive entropy in the fuzzy domain is defined as

$$e_a(\mu_X') = \frac{-1}{L \ln 2} \sum_{x=0}^{L-1} \left[\mu_X'(x) \ln(\mu_X'(x)) + (1 - \mu_X'(x) \ln(1 - \mu_X'(x)\right] P(x), x \le a$$

$$= \frac{-1}{L \ln 2} \sum_{x=0}^{L-1} (1 - \left[\mu_X'(x)) \ln((1 - \mu_X'(x)) + (1 - (1 - \mu_X'(x)) \ln(1 - (1 - \mu_X'(x))\right] P(x), x \succ a$$

$= \sum_{x \in X} E_a(f(x))$, where

a) $\mu_X^{'}(x)$ is the modified membership function corresponding to the inverse operator from the fuzzy domain to the spatial domain.

b) $\frac{1}{L\ln 2}$ is a constant that merely amounts the choice of a unit of measure which plays a key role in the approximation of change in image quality and entropy based change in image quality.

c) $\mu_X(x)$ represents the membership function of V component for a value x and x ranges from 0, 1, 2……,L-1.

d) The functional $E_a(f(x))$ is also satisfying the conditions in definition 4.2

Here we are using the additive Shannon's entropy for the case $x \le a$.

5. Proposed Algorithm

Let f be the image at one hand and is a degraded version of two valued image $g(i,j)$ representing light objects on a dark background. Our aim is to reconstruct $g^{'}$ from f by contrast minimization technique. This amounts to approximating f by minimizing a suitable contrast functional J_c (objective function). To each pixel position (i , j) we associate a contrast which is proportional to $g(i,j) - f(i,j)$. To keep approximating function g from fluctuating, we associate a cross over point μ_c and modified membership function such that $J_C = \left\| g^{'} - Eg \right\|^2 + \lambda(C_f - C_d)$. This approach actually produces a function $g^{'}(i,j)$ which is an improved version of $g(i,j)$. A minimization of entropy on J_c produce an effect on the approximating function $g^{'}(i,j)$ which is often looks enhanced. Here we choose the cross over point in the range 0.30 to 0.36. It can be seen from the experimental results and from the graphs that the approximated value of μ_c 0.32. Select the defuzzification function as defined in [1], $x^{'} = [-2\ln(\mu_X^{'}(x))\, f_h^2]$. Then the defuzzification function corresponding to the approximated μ_c is close as possible to a two valued function. So we seek an approximating function $g^{'}$ which is regarded as a fuzzy subset of the plane. Here μ_c is considered as a parameter supplied by the user. To enhance the image we choose the corresponding t-value also from 12 to 16 and produce a sequence of approximation to reach the minimum point. Corresponding to the parameters μ_c, t, f_h we calculate entropy defined as above so that the global contrast function is defined as $J_C = \left\| g^{'} - Eg \right\|^2 + \lambda(C_f - C_d)$ where λ is the Lagrange multiplier defined by the user. To reach the minimum point P (μ_c , t, f_h) we find the derivative of J_c and then apply it to the $(i+1)^{th}$ stage $P_{i+1} = P_i + \varepsilon S_i(\partial J_c / \partial P_i)$ and continue this process until the required approximation is obtained.

6. Algorithm for image enhancement

1. Use Gaussian membership function and covert RGB to HSV.
2. Use histogram to covert to Shannon entropy to fuzzy entropy.

3. Calculate the parameters of V for image enhancement.
4. Choose μ_c values from 0.30 to 0.36.
5. Modify the membership function corresponding to the approximate value of μ_c
6. Defuzzify V for the enhanced value.
7. Convert HSV to RGB and display.

7. Use of Lagrange multipliers in the Optimization function

In mathematical optimization problems the method of Lagrange multipliers named after Joseph Louis Lagrange is a method of finding the extrema of a function of several variables subject to one or more constraints. It is the basic tool in nonlinear constrained optimization. Lagrange multipliers compute the stationary points of the constrained function. By Fermat's theorem extrema occur either at these points or on the boundary or at points where the function is not differentiable. It reduces finding stationary points of a constrained function in n variables with k constraints to finding stationary of unconstrained function in n+k variables. This method introduces a new unknown scalar variable (called Lagrange multiplier) for each constrained and defines a new function (called the Lagrangian) in terms of the original function, the constraints and the Lagrange multipliers. The method of metric approximation is used to derive necessary conditions of optimality for general non smooth extremal problems. The main idea of the method is to approximate all constraints and functional by smooth distance like functions in order to obtain a smooth unconstrained optimization problem. Applying Fermat's theorem and taking limit in necessary conditions for the unconstrained problems, one can derive necessary conditions of optimality for the optimal extremal problem. In our discussion $J_C = \left\| g' - Eg \right\|^2 + \lambda(C_f - C_d)$ is a convex optimization problem. The error function E is the cross entropy.

8. Computational Complexity and Enhancement measure

In order to assess the image enhancement, we used the most well known image enhancement measure, the Tenengrad criterion (named after Tenenbaum and Schlag) [16], [17] to compare the results using fuzzy technique. When comparing an enhanced image with the original image it is observed that the original image lacks high frequencies (or widths). The Tenengrad criterion is based on gradient magnitude maximization method which was the most robust functionally accurate image quality measures. According to paper [16] the advantages of the Tenengrad criterion as:

1. The measured sharpness varies monotonically as the object moved towards the position of best enhancement.
2. The criterion shows a strong and sharp response at the position of best enhancement.
3. The criterion is robust to low contrast to noise ratio.
4. It is very easy to implement and has linear complexity.

This criterion as well as most other enhancement measure criterions are sensitive to contrast induced by image acquisition as well as variations in image enhancement. The Tenengrad value of the image I can be calculated from the gradient $\nabla f(x, y)$ at each pixel (x, y) where the derivatives are obtained by a high pass filter Here the Sobel operator with the convolution kernels i_x and i_y has been used in Tenengrad measure and the gradient magnitude results to $S(x,y)=\sqrt{(i_x * I(x,y)^2+(i_y * I(x,y)^2}$ and the final criterion for a given enhancement position z is defined as $C(z)=\sum_x\sum_y S(x,y)^2$ for $S(x,y)\succ T$, where is T is a threshold. Usually the image quality is higher if Tenengrad value is larger. We calculated the Tenengrad values (TEN) of all images in this article, and listed them in Table-II. It is clear from the enhanced images that the Tenengrad values are higher than the original image. This result also agrees with the visual evaluation by the human eye. Therefore the conditions in theorem 3.1is satisfied. Hence $g^{'}$ approximates f .

9. Conclusions

Table 1. Image Enhancement result for Approximate Value of μ_c

	t		μ_c		f_h	
	Initial	**Enhanced**	**Initial**	**Enhanced**	**Initial**	**Enhanced**
Boy	5	12.4068	0.3	0.3119	143.1898	143.1983
Girl	5	14.6042	0.3	0.3141	150.5285	150.5384
Light house	5	16.8515	0.3	0.3122	142.5704	142.5792
shoe	5	13.6113	0.3	0.3142	155.4743	155.4840
Building	5	14.1133	0.3	0.3105	157.0967	157.1034

Table 2.

Images	**Tenengrad of original image**	**Tenengrad of enhanced image**
Boy	26230	34169
Girl	24595	70880
Light house	22006	49339
shoe	23474	74288
Building	62644	88810

This paper proposed an approximate value for μ_c (shown in table I and its corresponding graph) that effects the approximation of evaluation mapping on different types of images. We have considered five different types of images and the graph shown that there is an approximation of μ_c between 0.30 and 0.36. The lattice theory and partially ordered set shows that one can choose any membership value between 0 and $1/2$ for under exposed images. Therefore one can shrink the approximate mapping function to the sub interval in the compact domain $I=[0,1]$ into [0, 1/2]. The inverse problem of image enhancement discussed in section 3, gives a clear idea about the entropy measure and the regularization term. In this paper $\lambda(C_f-C_d)$ is used as the regularization term to avoid the ill conditioning of the problem. The iterative process can start with $P_{i+1}=P_i+\varepsilon S_i(\partial J_c/\partial P_i)$ till the best approximated value is obtained .Thus the image is enhanced.

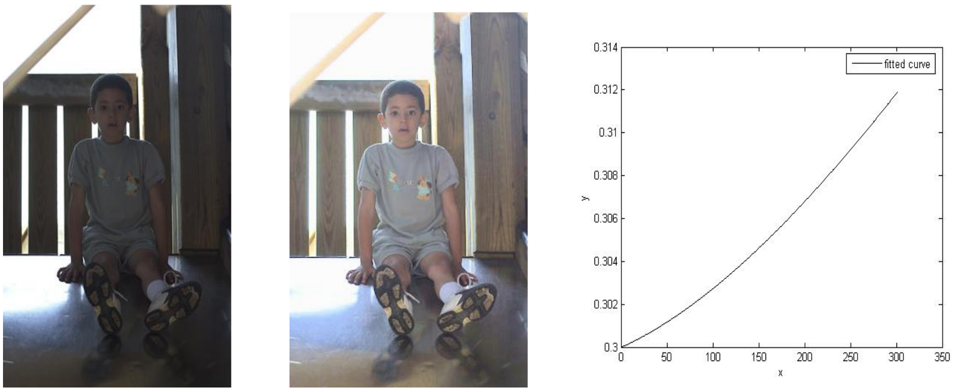

Figure 1. Original boy, Enhanced boy, Boy μ_c Value

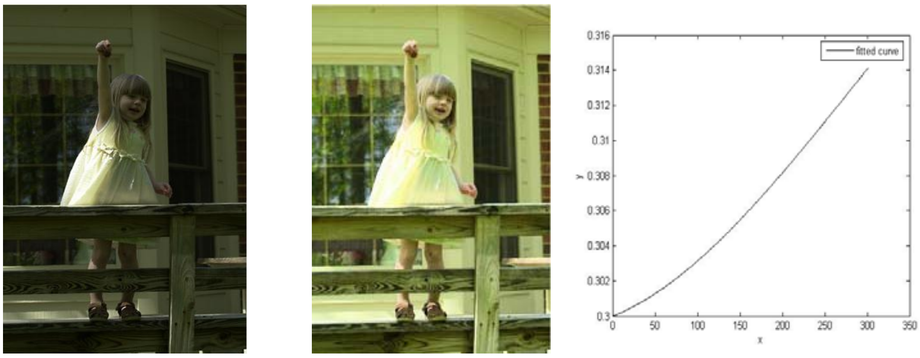

Figure 2. Original Girl, Enhanced Girl, Girl μ_c Value

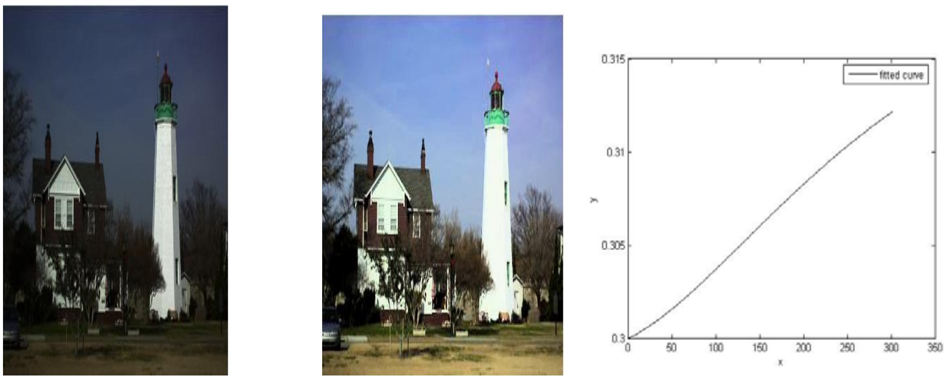

Figure 3. Original Light House , Enhanced Light House, Light house μ_c Value

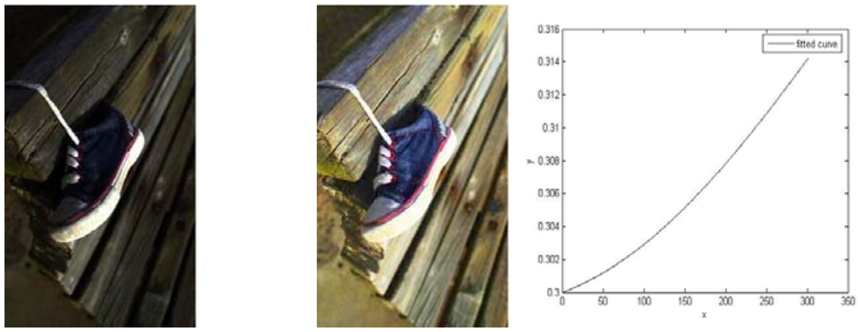

Figure 4. Original Shoe, Enhanced Shoe , Shoe μ_c Value

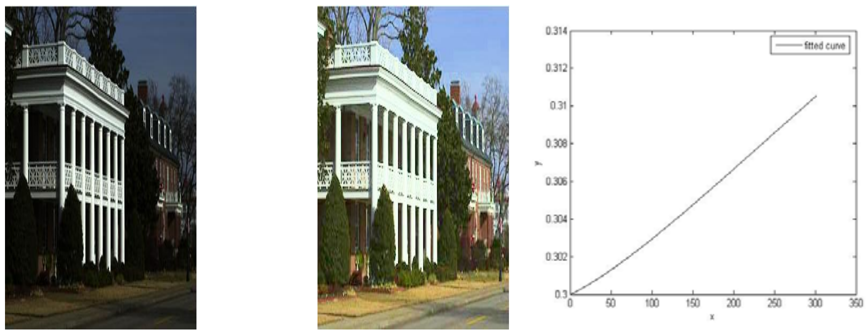

Figure 5. Original building, Enhanced building, Building μ_c Value

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